



GRADE 4 MATH: FARMER FRED

UNIT OVERVIEW

This approximately 4-week unit centers around understanding fractional parts of a whole and using fractions and mixed numbers to solve problems.

TASK DETAILS

Task Name: Farmer Fred

Grade: 4

Subject: Mathematics

Depth of Knowledge: 3

Task Description: Students use fractional parts of a whole, properties of shapes, congruency, and computation using fractions and money to determine the fair value and total worth of eight fields.

Standards:

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

4.NF.3c Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

4.NF.3d Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4c Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

Standards for Mathematical Practice:

MP.1 Make sense of problems and persevere in solving them.

MP.3 Construct viable arguments and critique the reasoning of others.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.



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The task and instructional supports in the following pages are designed to help educators understand and implement tasks that are embedded in Common Core-aligned curricula. While the focus for the 2011-2012 Instructional Expectations is engaging students in Common Core-aligned culminating tasks, it is imperative that the tasks are embedded in units of study that are also aligned to the new standards. Rather than asking teachers to introduce a task into the semester without context, this work is intended to encourage analysis of student and teacher work to understand what alignment looks like. We have learned through this year's Common Core pilots that beginning with rigorous assessments drives significant shifts in curriculum and pedagogy. Universal Design for Learning (UDL) support is included to ensure multiple entry points for all learners, including students with disabilities and English language learners.

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Acknowledgements: The unit outline was developed by David Graeber (CFN 534) with input from Curriculum Designers Alignment Review Team. The tasks were developed by the schools in the 2010-2011 NYC DOE Elementary School Performance Based Assessment Pilot, in collaboration with Exemplars, Inc. and Center for Assessment.

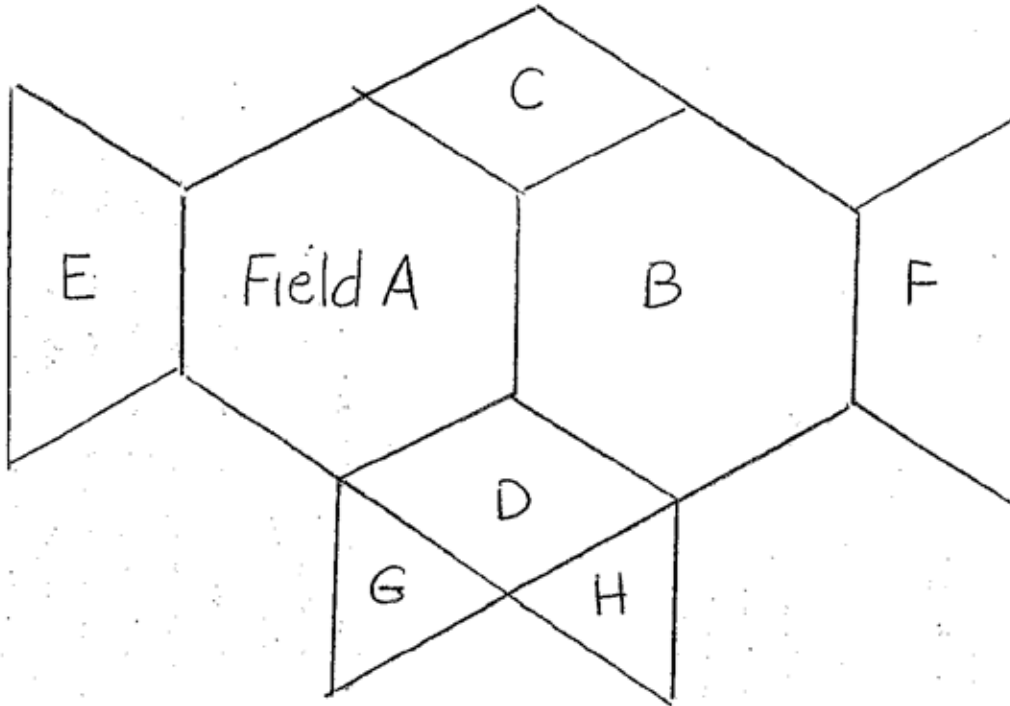


GRADE 4 MATH: FARMER FRED PERFORMANCE TASK

Name _____

Farmer Fred

Farmer Fred's fields are worth twelve hundred dollars total. The fields are formed with the same properties as your pattern blocks. Each field's value is based on its size. What fraction of the total value is each field worth? How much is each field worth? Show and explain all of your mathematical thinking.





GRADE 4 MATH: FARMER FRED

UNIVERSAL DESIGN FOR LEARNING (UDL) PRINCIPLES

Math Grade 4 – Farmer Fred
Common Core Learning Standards/
Universal Design for Learning

The goal of using Common Core Learning Standards (CCLS) is to provide the highest academic standards to all of our students. Universal Design for Learning (UDL) is a set of principles that provides teachers with a structure to develop their instruction to meet the needs of a diversity of learners. UDL is a research-based framework that suggests each student learns in a unique manner. A one-size-fits-all approach is not effective to meet the diverse range of learners in our schools. By creating options for how instruction is presented, how students express their ideas, and how teachers can engage students in their learning, instruction can be customized and adjusted to meet individual student needs. In this manner, we can support our students to succeed in the CCLS.

Below are some ideas of how this Common Core Task is aligned with the three principles of UDL; providing options in representation, action/expression, and engagement. As UDL calls for multiple options, the possible list is endless. Please use this as a starting point. Think about your own group of students and assess whether these are options you can use.

REPRESENTATION: *The “what” of learning.* How does the task present information and content in different ways? How students gather facts and categorize what they see, hear, and read. How are they identifying letters, words, or an author's style?

In this task, teachers can...

- ü **Pre-teach vocabulary and symbols, especially in ways that build a connection to the learners’ experience and prior knowledge** by providing text based examples and illustrations of fields. Integrate numbers and symbols into word problems.

ACTION/EXPRESSION: *The “how” of learning.* How does the task differentiate the ways that students can express what they know? How do they plan and perform tasks? How do students organize and express their ideas?

In this task, teachers can...

- ü **Anchor instruction by pre-teaching critical prerequisite concepts** through demonstration or models (i.e. use of two dimensional representations of space and geometric models).

ENGAGEMENT: *The “why” of learning.* How does the task stimulate interest and motivation for learning? How do students get engaged? How are they challenged, excited, or interested?

In this task, teachers can...

- ü **Optimize relevance, value and authenticity** by designing activities so that learning outcomes are authentic, communicate to real audiences, and reflect a purpose that is clear to the participants.

Visit <http://schools.nyc.gov/Academics/CommonCoreLibrary/default.htm> to learn more information about UDL.



GRADE 4 MATH: FARMER FRED

RUBRIC

The following section contains two rubrics that were used to score student work: a content rubric and a process rubric. The content rubric describes student performance according to the content standards in the CCLS. The process rubric describes student performance according to the National Council of Teachers of Mathematics (NCTM) process standards. Students' were given a score based on their achievement on the CCLS content rubric and the process rubric. Given that the process rubric is not in the language of the Common Core's Mathematical Practices, we have also included a document that NCTM has posted regarding the relationship between the NCTM process standards and the Standards for Mathematical Practice.

CCSS Mathematics Content Standards Rubric – Grades 3 & 4

Students apply mathematical concepts, reasoning, and procedural skills in problems-solving situations and support their solutions using computations, mathematical language, and appropriate representations/ modeling.

CCSS Math Criteria by Strand	Novice	Apprentice	Practitioner	Expert
Number & Operations in Base Ten , Number & Operations – Fractions, & Operations & Algebraic Thinking	<p>Applies flawed strategies (e.g., attempts to form groups when multiplying, but does not use equal sized groups)</p> <p>Selects the incorrect operation to perform or major inaccuracies in computation lead to an incorrect solution</p> <p>Still demonstrates limited knowledge of place value or number sense (e.g., difficulty estimating, representing part-whole relationships; cannot determine reasonableness of an answer; does not see relationship between multiplication-division)</p> <p>A correct answer may be stated, but is not supported by student work or explanations</p>	<p>Some parts of problem correct and those parts are supported by student work</p> <p>Uses visual models (number line, area models, set models) to represent and compare parts of a whole, but stops short of applying concepts in problem solving contexts</p> <p>Demonstrates relationships between multiplication and division with whole numbers using number facts, objects, visuals, and symbols/equations</p> <p>May include limited/ partial explanations for solutions</p> <p>Uses 4 operations with small whole numbers, but displays some inaccuracies in computations of large numbers (multi-digit) and small numbers (fractions, decimals)</p>	<p><i>Expresses whole numbers as fractions (gr 3) and fractions as decimals with 10 or 100 as denominator (gr 4)</i> 3.NF-3 4.NF-5, 6</p> <p><i>Uses place value to round whole numbers</i> 3.NBT-1 4.NBT-3</p> <p><i>Expresses fractions and equivalent fractions (gr 3-4) and decimal-fraction equivalents (gr 4); explains/ illustrates why they are/are not equivalent (e.g., number lines, area models, sets; compare to benchmarks)</i> 3.NF-1, 2, 3 4.NF-1, 2, 5, 6, 7</p> <p><i>Uses addition, subtraction, and multiplication to solve problems with fractions (gr3-4) and mixed numbers (gr 4)</i> 3.NF-1, 2, 3 4.NF-3, 4</p> <p><i>Uses 4 operations in solving multi-step problems and word problems with whole numbers (e.g., using equations, arrays, explaining patterns using whole numbers, following a rule)</i> 3.OA-3, 4, 5, 6, 7, 8, 9 3.NBT-2, 3 4.OA-2, 3, 5</p> <p><i>May be some minor flaws when performing multi-step computations, but <u>procedural and conceptual understanding is clearly evident</u></i></p>	<p>All parts of problem correct, precise, and supported by student work or explanations</p> <p>Extends understanding of equivalence of fractions by identifying proper and improper fractions</p> <p>Interprets meaning of the products when multiplying (gr 3-4) and remainders when dividing (gr 4)</p> <p>Uses a variety of representations (e.g., concrete models, diagrams, equations), strategies (e.g., place value, properties of operations), and algorithms to solve problems or represent solutions in multiple ways</p> <p>Uses additive or multiplicative reasoning to solve or interpret most problems</p>

NOTE: Anchor papers will illustrate how descriptors for each performance level are evidenced at each grade.

Exemplars[®] Standards-Based Math Rubric^{*}

	Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Novice	<p>No strategy is chosen, or a strategy is chosen that will not lead to a solution.</p> <p>Little or no evidence of engagement in the task present.</p>	<p>Arguments are made with no mathematical basis.</p> <p>No correct reasoning nor justification for reasoning is present.</p>	<p>No awareness of audience or purpose is communicated.</p> <p>or</p> <p>Little or no communication of an approach is evident</p> <p>or</p> <p>Everyday, familiar language is used to communicate ideas.</p>	<p>No connections are made.</p>	<p>No attempt is made to construct mathematical representations.</p>
Apprentice	<p>A partially correct strategy is chosen, or a correct strategy for only solving part of the task is chosen.</p> <p>Evidence of drawing on some previous knowledge is present, showing some relevant engagement in the task.</p>	<p>Arguments are made with some mathematical basis.</p> <p>Some correct reasoning or justification for reasoning is present with trial and error, or unsystematic trying of several cases.</p>	<p>Some awareness of audience or purpose is communicated, and may take place in the form of paraphrasing of the task.</p> <p>or</p> <p>Some communication of an approach is evident through verbal/ written accounts and explanations, use of diagrams or objects, writing, and using mathematical symbols.</p> <p>or</p> <p>Some formal math language is used, and examples are provided to communicate ideas.</p>	<p>Some attempt to relate the task to other subjects or to own interests and experiences is made.</p>	<p>An attempt is made to construct mathematical representations to record and communicate problem solving.</p>

^{*}Based on revised NCTM standards.

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Exemplars[®] Standards-Based Math Rubric (conf.)*

	Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Practitioner	<p>A correct strategy is chosen based on mathematical situation in the task.</p> <p>Planning or monitoring of strategy is evident.</p> <p>Evidence of solidifying prior knowledge and applying it to the problem solving situation is present.</p> <p>Note: The practitioner must achieve a correct answer.</p>	<p>Arguments are constructed with adequate mathematical basis.</p> <p>A systematic approach and/or justification of correct reasoning is present. This may lead to...</p> <ul style="list-style-type: none"> • clarification of the task. • exploration of mathematical phenomenon. • noting patterns, structures and regularities. 	<p>A sense of audience or purpose is communicated.</p> <p>and/or</p> <p>Communication of an approach is evident through a methodical, organized, coherent sequenced and labeled response.</p> <p>Formal math language is used throughout the solution to share and clarify ideas.</p>	<p>Mathematical connections or observations are recognized.</p>	<p>Appropriate and accurate mathematical representations are constructed and refined to solve problems or portray solutions.</p>
Expert Work at this level is exceeding grade-level expectations	<p>An efficient strategy is chosen and progress towards a solution is evaluated.</p> <p>Adjustments in strategy, if necessary, are made along the way, and / or alternative strategies are considered.</p> <p>Evidence of analyzing the situation in mathematical terms, and extending prior knowledge is present.</p> <p>Note: The expert must achieve a correct answer.</p>	<p>Deductive arguments are used to justify decisions and may result in formal proofs.</p> <p>Evidence is used to justify and support decisions made and conclusions reached. This may lead to...</p> <ul style="list-style-type: none"> • testing and accepting or rejecting of a hypothesis or conjecture. • explanation of phenomenon. • generalizing and extending the solution to other cases. 	<p>A sense of audience and purpose is communicated.</p> <p>and/or</p> <p>Communication at the Practitioner level is achieved, and communication of argument is supported by mathematical properties.</p> <p>Precise math language and symbolic notation are used to consolidate math thinking and to communicate ideas.</p>	<p>Mathematical connections or observations are used to extend the solution.</p>	<p>Abstract or symbolic mathematical representations are constructed to analyze relationships, extend thinking, and clarify or interpret phenomenon.</p>

*Based on revised NCTM standards.

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National Council of Teachers of Mathematics (NCTM) Process Standards and the Common Core State Standards for Mathematics

From *NCTM Action on the Common Core State Standards for Mathematics*

by NCTM President J. Michael Shaughnessy

"The preeminent message in both the NCTM *Principles and Standards for School Mathematics* (2000) and CCSSM is the importance of nurturing mathematical thinking and reasoning processes in our students. No bulleted list of specific content standards will hold together as a coherent, meaningful whole, or make any significant contribution to our students' growth in mathematics, without interweaving mathematical "practices." Mathematics curricula must show students the power of reasoning and sense making as they explore mathematical structures, of communication as they construct viable arguments, and of multiple representations as they engage in mathematical modeling. The close connections between the NCTM Process Standards and the CCSSM Standards for Mathematical Practice are represented in the chart below.

The upcoming NCTM publication, *Making it Happen*, will provide a deeper analysis of the connections between the NCTM Process Standards and detail the potential of the existing NCTM resources to interpret and implement CCSSM."

NCTM Process Standards and the CCSS Mathematical Practices	
NCTM Process Standards	CCSS Standards for Mathematical Practice
Problem Solving	1. Make sense of problems and persevere in solving them. 5. Use appropriate tools strategically.
Reasoning and Proof	2. Reason abstractly and quantitatively. 3. Critique the reasoning of others. 8. Look for and express regularity in repeated reasoning
Communication	3. Construct viable arguments
Connections	6. Attend to precision. 7. Look for and make use of structure
Representations	4. Model with mathematics.



GRADE 4 MATH: FARMER FRED

ANNOTATED STUDENT WORK

This section contains annotated student work at a range of score points, student summaries, and implications for instruction for each performance level. The annotated student work and student summaries demonstrate performance at different levels and show examples of student understandings and misunderstandings of the task that can be used with the implications for instruction to understand how to move students to the next performance level.

Grade 4 Math: Farmer Fred Annotated Student Work: Expert

This student is an Expert according to both the Exemplars Rubric and the CCSS Content Standards Rubric, (both included in the supporting materials).

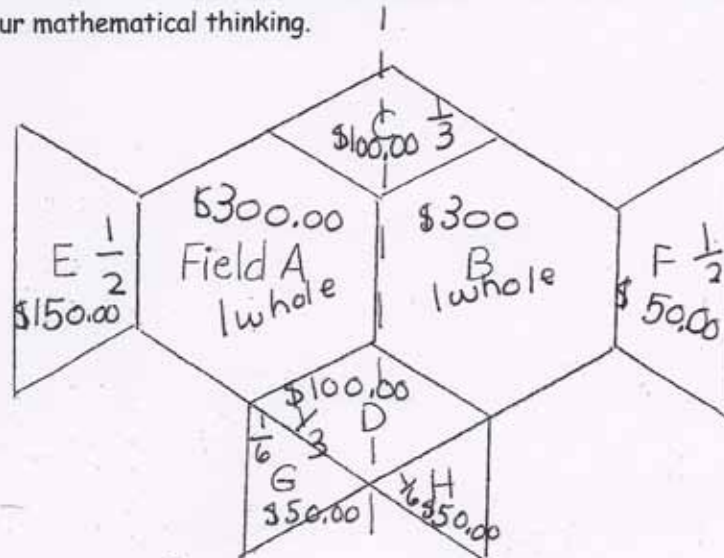
Expert, Student 1

Grade 4

Name _____

Farmer Fred

Farmer Fred's fields are worth twelve hundred dollars total. The fields are formed with the same properties as your pattern blocks. Each field's value is based on its size. What fraction of the total value is each field worth? How much is each field worth? Show and explain all of your mathematical thinking.



I have to figure out what the fields are worth and the fraction of the fields. You can't solve this problem if you don't know fractions. I will make a table and write the answers on it.

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Grade 4 Math: Farmer Fred Annotated Student Work: Expert

The student correctly extends understanding of fraction equivalence and ordering (4.NF 1, 2), builds fractions from unit fractions by applying and extending previous understandings of operations on fractions (4.NF 3 a, b, d), and uses operations with whole numbers to solve the second part of the problem (4.OA 2). The student also brings an understanding of symmetry to the problem (4.G), decimals (4.NF 6), area (4.MD), and percents.

The student models mathematics by using a table and diagrams to indicate the fields, their fractional value and worth. All labels and data are correct. The student uses these results to extend thinking to decimals, percents, and area.

The student uses precise mathematical language to support her/his viable argument. Some terms include the names of the shapes, area, and fraction, money, decimal, and percent notation.

everything has to be equivalent
two trapezoids make one hexagon
two triangles make a rhombus
so the diagram really has 4 hexagons

So $4 \overline{)1200} 300.00$ now you can do all the shapes
 $150 + 150 = 300$ $100 + 100 + 100 = 300$
 $50 + 50 + 50 + 50 + 50 + 50 = 300$

the 3, 6, 2 are the denominators

$\frac{100}{300}$ $\frac{50}{300}$ $\frac{150}{300}$ $\frac{100}{300}$ $\frac{50}{300}$

Fred's Fields

Field	fractions	field values
A	1 whole $\frac{1}{6}$	\$300.00
B	1 whole $\frac{1}{6}$	\$300.00
C	$\frac{1}{3}$	\$100.00
D	$\frac{1}{3}$	\$100.00
E	$\frac{1}{2}$	\$150.00
F	$\frac{1}{2}$	\$150.00
G	$\frac{1}{6}$	\$50.00
H	$\frac{1}{6}$	\$50.00

This is how it works
 $\triangle + \triangle = \square$
 $\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$
 $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1$
 $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$

My connections List - I thought of 7

1. The diagram has 1 line of symmetry I put it in.
2. shape names \square hexagon \diamond rhombus \triangle triangle ∇ trapezoid
3. I know some percents \square 100% ∇ 50%
4. I know some decimals \square 1.00 ∇ .50 or .50
5. There are really 24 \triangle 's in the diagram or 24 sixths
 So $\frac{24}{6} = 4$ \square or $24 \overline{)1200} 50.00$ this way you multiply 2, 6, 3 to get the \square \diamond \triangle values, I am correct.
6. ∇ 2 trapezoids have same area as \square hexagon
7. You can't have a square field - no equivalent area

2x 50 = 100 \diamond
 3x 50 = 150 ∇
 6x 50 = 300 \square

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The student is able to make sense and persevere in solving the problem. The student applies correct understanding to determine the fractional value and worth of each field. The student is able to step back and provide a new perspective to the problem by finding the line of symmetry. The student reasons abstractly and quantitatively by extending her/his thinking to decimals, and area. The student evaluates the reasonableness of her/his answer by applying a different property of operation.

The student looks for and makes use of structure and interprets mathematical results in the mathematics model to evaluate the reasonableness of her/his results by verifying her/his answer by using twenty-four triangles to determine four whole hexagons and stating, "I am correct." The student also brings the understanding of percents, decimals and area to her/his solution.

Grade 4 Math: Farmer Fred Annotated Student Work: Expert

Expert – Student 1 Summary

Achievement Level: Student 1 is an Expert according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Expert	The student's strategy of creating a table to show each field, determining the fractional part of each field and the worth of each field works to solve the problem. The student labels the second column, "answers fractions," and the third column, "answers field values." The student's answer, "Field A and B as 1 whole, $\frac{6}{6}$ and \$300.00, Field C and D as $\frac{1}{3}$ and \$100.00, Field E and F as $\frac{1}{2}$ and \$150.00, Field G and H as $\frac{1}{6}$ and \$50.00," is correct. The student extends her/his thinking to fractions, decimals, and area. The student also uses different reasoning to verify her/his answer is correct.
Reasoning and Proof Expert	The student demonstrates correct reasoning of the underlying mathematical concepts in the problem. The student applies understanding of fractional parts of a whole. The student determines that all the field shapes can form four hexagons and divides 1200 by 4 for a quotient of 300. The student divides 300 by the denominators 3, 6, and 2 to find the remaining worth of the trapezoid, rhombus and triangle shaped fields. The student draws in the line of symmetry, compares fractions to decimals and percents and uses the concept of area to determine why a field cannot be shaped as a square in this problem. the student also verifies her/his answer by dividing 1200 by 24 (triangles), finds the correct worth of the fields and states, "I am correct again."
Communication Expert	The student correctly uses the mathematical terms-worth, fractions, values, from the problem. The student also correctly uses the terms-equivalent, trapezoids, hexagon, triangles, rhombus, shapes, denominators, whole, line of symmetry, percents, decimals, sixths. The student correctly uses the mathematical notation- $\frac{6}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{6}$, $\frac{24}{6}$, $\frac{2}{2}$, $\frac{3}{3}$, $\frac{24}{6}$, 100%, 50%, 1.00, .5, .50, \$300.00, \$100.00, \$150.00, \$50.00.

Connections Expert	<p>The student makes the mathematically relevant Practitioner connections, "the 3, 6, 2 are denominators," "The diagram has 1 line of symmetry. I put it in," and, "shape names-hexagon, rhombus, trapezoid, triangle." The student makes the Expert connection by verifying that her/his answer is correct. The student states, "There are really 24 ▲'s in the diagram or 24 sixths. So $24/6 = 4$." The student then divides 1200 by 24 for a quotient of 50 and states, "This way you multiply 2, 6, 3, to get the trapezoid, hexagon, rhombus values." The student does the computation on her/his paper and states, "I am correct." The student also states, "I know some percents. Hexagon 100%, trapezoid 50%," "I know some decimals. hexagon 1.00, trapezoid .5 or .50." The student considers area and states, "2 trapezoids have same area as 1 hexagon. You can't have a square field-no equivalent area."</p>
Representation Expert	<p>The student's table is appropriate and accurate with all columns correctly labeled and all entered data is correct. The diagrams the student uses in her/his solution are appropriate and accurate. The student labels all the shapes correctly. The student uses the table and diagrams to support her/his thinking of how to solve the problem a different way to verify her/his answer and to explore area.</p>

Instructional Implications: Farmer Fred, 4

Achievement Level: Expert

Note: Student work identified at this level is exceeding grade-level expectations

The following is a list of instructional implications that you may want to consider for students performing at the Expert level. In addition, you may want to consult the suggestions for the Practitioner level:

- Solve problem more than one way to verify that the answer is correct and link the two strategies together
- Relate problem to a similar one completed and discuss how they are mathematically similar
- Use percents to define the value of each field
- Use decimals to define the value of each field

Provide a total value of all the fields that would require the student to work with decimals/cents to find the worth of each field

Find the line of symmetry in the field

Explain why a field is not in the shape of the orange pattern block (square) or the thin tan pattern block (rhombus)-explore angles, length of side, etc.

Grade 4 Math: Farmer Fred
Annotated Student Work: Practitioner

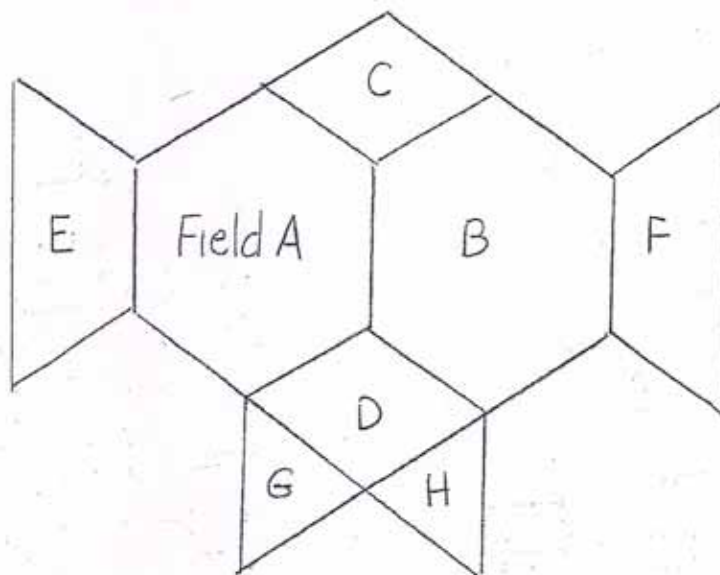
Practitioner, Student 1

Name _____

This student is a Practitioner according to both the Exemplars Rubric and the CCSS Content Standards Rubric (both included in the supporting materials).

Farmer Fred

Farmer Fred's fields are worth twelve hundred dollars total. The fields are formed with the same properties as your pattern blocks. Each field's value is based on its size. What fraction of the total value is each field worth? How much is each field worth? Show and explain all of your mathematical thinking.



I need to find out how much and what fraction each field is worth. I will use a chart for both problems.

The student is able to make sense and persevere in solving the problem. The student applies understanding of fractional parts of a whole in determining the correct fractional value of each field and applies correct number and operations thinking to find the correct worth of each field.

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Grade 4 Math: Farmer Fred Annotated Student Work: Practitioner

The student correctly extends understanding of fraction equivalence and ordering (4. NF 1, 2), builds fractions from unit fractions by applying and extending previous understandings of operations on fractions (4.NF 3 a, b, d), and uses operations with whole numbers to solve the second part of the problem (4.OA 2).

The student models with mathematics by using a table to indicate the fields, their worth, and fractional value. All labels and data are correct.

Each field is worth and fraction answers

Field	worth	fraction of field
A	\$300.00	$1 \text{ or } \frac{1}{6}$
B	\$300.00	$1 \text{ or } \frac{1}{6}$
C	\$100.00	$\frac{1}{3} \text{ or } \frac{2}{6}$
D	\$100.00	$\frac{1}{3} \text{ or } \frac{2}{6}$
E	\$150.00	$\frac{1}{2} \text{ or } \frac{3}{6}$
F	\$150.00	$\frac{1}{2} \text{ or } \frac{3}{6}$
G	\$50.00	$\frac{1}{6}$
H	\$50.00	$\frac{1}{6}$

One Connection I will like to make is that you can convert the fractions in the problem. For Ex: $\frac{1}{2}$ can be $\frac{4}{8}$ and still be the same value. Another connection I will like to do is that A and B is a hexagon. And I know that C and D are Rhombi and that E, F are trapezoids and G, H are triangles. The 8 fields use 4 hexagons. You can't do this problem if you don't know the pattern blocks. Like $\frac{6}{6} = 1$ or $\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$. I started with the triangle and added up by 100. It didn't work so I used 50 and it worked. I added and times in my head. It wasn't very hard because of the zeros.

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The student uses precise mathematical language to support her/his viable argument. Some terms include the names of the shapes, fraction and money notation.

The student reasons quantitatively by making the connection, "You can convert the fractions in the problem...."

Grade 4 Math: Farmer Fred Annotated Student Work: Practitioner

Practitioner – Student 1 Summary

Achievement Level: Student 1 is a Practitioner according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Practitioner	The student's strategy of creating a table to show each field, using guess and check to find each field's worth, and determining the fractional part of each shape to the hexagon works to solve the problem. The student writes, "Each field is worth and fraction answers," above her/his table. The student's answer, "Field A and B as \$300.00 and $\frac{6}{6}$, Field C and D as \$100.00 and $\frac{1}{3}$ or $\frac{2}{6}$, Field E and F as \$150.00 and $\frac{1}{2}$ or $\frac{3}{6}$, and Field G and H as \$50.00 and $\frac{1}{6}$," is correct.
Reasoning and Proof Practitioner	The student demonstrates correct reasoning of the underlying mathematical concepts in the problem. The student applies understanding of fractional parts of a whole when considering the "hexagon," trapezoid, rhombus, and triangle shapes and determines what fraction of the total value each field is worth using sixths and equivalent fractions. The student also states correct reasoning in determining the correct worth of each field.
Communication Practitioner	The student correctly uses the mathematical terms-fraction, worth, value, from the task. The student also correctly uses the terms-chart, hexagon, rhombi, "trapizoids," triangles, shapes. The student correctly uses the mathematical notation- $\frac{6}{6}$, $\frac{1}{3}$, $\frac{2}{6}$, $\frac{1}{2}$, $\frac{3}{6}$, \$300.00, \$100.00, \$150.00, \$50.00.
Connections Practitioner	The student makes the mathematically relevant connections, "You can convert the fractions in the problem For Exm: $\frac{1}{2}$ can be $\frac{4}{8}$ and still be the same value," "A and B is a hexagon," "C and D are Rhombi," "E, F are trapizoids," "G, H are triangles," and, "The 8 fields use 4 hexagon shapes."
Representation Practitioner	The student's table is appropriate and accurate with all columns correctly labeled and all entered data is correct.

Grade 4 Math: Farmer Fred

Annotated Student Work: Practitioner

This student is a Practitioner according to both the Exemplars Rubric and the CCSS Content Standards Rubric (both included in the supporting materials).

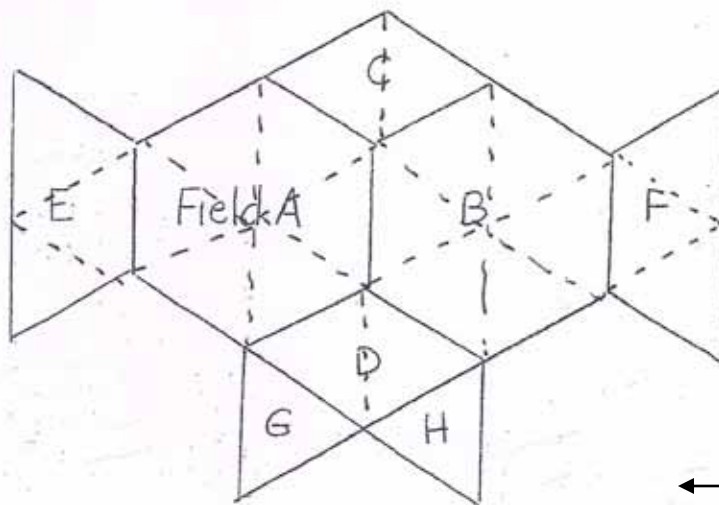
Practitioner, Student 2

Grade 4

Name _____

Farmer Fred

Farmer Fred's fields are worth twelve hundred dollars total. The fields are formed with the same properties as your pattern blocks. Each field's value is based on its size. What fraction of the total value is each field worth? How much is each field worth? Show and explain all of your mathematical thinking.



I will find out the fraction and how much value each field is. I have to find how many sixths there are. I will use pattern blocks to figure it out and the diagram.

The student is able to make sense and persevere in solving the problem. The student applies understanding of fractional parts of a whole in determining the correct fractional value of each field and applies correct number and operations thinking to find the correct worth of each field.

Page 1 of 2

Grade 4 Math: Farmer Fred Annotated Student Work: Practitioner

The student uses precise mathematical language to support her/his viable argument. Some terms include the names of the shapes, fraction and money notation.

$$\begin{array}{r} \$50.00 \\ 24 \overline{) \$200.00} \\ \underline{120} \\ 0 \end{array}$$

$$\begin{array}{r} 24 \\ \times 5 \\ \hline 120 \end{array}$$

$$\begin{array}{r} 50 \\ \times 6 \\ \hline 300 \end{array} \quad \begin{array}{r} 50 \\ \times 2 \\ \hline 100 \end{array}$$

The student correctly extends understanding of fraction equivalence and ordering (4.NF 1, 2), builds fractions from unit fractions by applying and extending previous understandings of operations on fractions (4.NF 3 a, b, d), and uses operations with whole numbers to solve the second part of the problem (4.OA 2).

the fields			
field	shape	value	fraction
A	hexagon	\$300.00	$\frac{1}{1}$
B	hexagon	\$300.00	$\frac{1}{1}$
C	rhombus	\$200.00	$\frac{1}{3}$
D	rhombus	\$200.00	$\frac{1}{3}$
E	trapezoid	\$150.00	$\frac{1}{2}$
F	trapezoid	\$150.00	$\frac{1}{2}$
G	triangle	\$50.00	$\frac{1}{6}$
H	triangle	\$50.00	$\frac{1}{6}$

$$\begin{array}{r} 50 \\ \times 3 \\ \hline 150 \end{array}$$

$$\begin{array}{r} 600 \\ 200 \\ 300 \\ \hline 1200 \end{array}$$

The student models with mathematics by using a table to indicate the fields, their shape, fractional value, and worth. All labels and data are correct.

answer 2

answer 1

Connections

I named the shapes

I see $\frac{24}{6}$ equals 4 hexagons



24 triangles
is 4 hexagons

The student reasons quantitatively by making the connection, "I see $\frac{24}{6}$ equals 4 hexagons."

page 2 of 2

Grade 4 Math: Farmer Fred Annotated Student Work: Practitioner

Practitioner – Student 2 Summary

Achievement Level: Student 2 is a Practitioner according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Practitioner	The student's strategy of creating a table to show each field, the name of the shape of each field, determining the fair value of each field, and the fractional part of each shape to the hexagon works to solve the problem. The student directs arrows to the third and fourth columns to indicate her/his answer. The student's answer, "Field A and B as \$300.00 and $1/1$, Field C and D as \$100.00 and $1/3$, Field E and F as \$150.00 and $1/2$, and Field G and H as \$50.00 and $1/6$," is correct.
Reasoning and Proof Practitioner	The student demonstrates correct reasoning of the underlying mathematical concepts in the problem. The student applies understanding of fractional parts of a whole. The student uses the fractional value of the triangle, $1/6$, to determine what fraction of the total value each field is worth. The student divides \$1200.00 by the twenty-four triangles found in the 8 fields. The student multiplies the quotient, \$50.00, by the number of triangles there are in a hexagon, trapezoid, and rhombus.
Communication Practitioner	The student correctly uses the mathematical terms—fraction, value—from the task. The student also correctly uses the terms—sixths, diagram, shape, hexagon, rhombus, trapezoid, triangle, equal. The student correctly uses the mathematical notation— $1/1$, $1/3$, $1/2$, $1/6$, $24/6$, \$300.00, \$100.00, \$150.00, \$50.00.
Connections Practitioner	The student makes the mathematically relevant connections, "I named the shapes," "I see $24/6$ ▲ equals 4 hexagons," and, "24 triangles is 4 hexagons."
Representation Practitioner	The student's table is appropriate and accurate with all columns correctly labeled and all entered data is correct. The student's diagram is appropriate and accurate. The student defines the diagram by stating, "24 triangles is 4 hexagons."

Grade 4 Math: Farmer Fred
Annotated Student Work: Practitioner

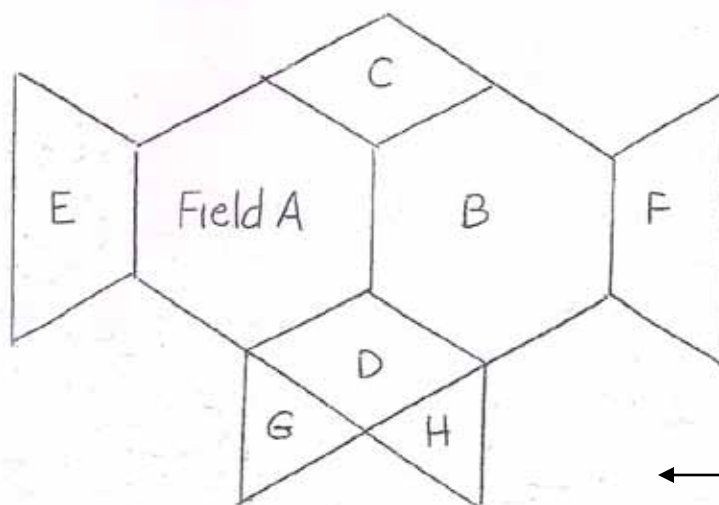
Practitioner, Student 3

Name _____

This student is a Practitioner according to both the Exemplars Rubric and the CCSS Content Standards Rubric, (both included in the supporting materials).

Farmer Fred

Farmer Fred's fields are worth twelve hundred dollars total. The fields are formed with the same properties as your pattern blocks. Each field's value is based on its size. What fraction of the total value is each field worth? How much is each field worth? Show and explain all of your mathematical thinking.



I am going to find out how much each field is worth and the fraction value. I will make a chart

The student is able to make sense and persevere in solving the problem. The student applies understanding of fractional parts of a whole in determining the correct fractional value of each field and applies correct number and operations thinking to find the correct worth of each field.

page 1 of 2

Grade 4 Math: Farmer Fred Annotated Student Work: Practitioner

The student correctly extends understanding of fraction equivalence and ordering (4. NF 1, 2), builds fractions from unit fractions by applying and extending previous understandings of operations on fractions (4. NF 3 a, b, d), and uses operations with whole numbers to solve the second part of the problem (4. OA 2).

fields	field fraction	field value
E	$\frac{1}{2}$	\$150.00
F	$\frac{1}{2}$	\$150.00
G	$\frac{1}{6}$	\$50.00
H	$\frac{1}{6}$	\$50.00
C	$\frac{1}{3}$	\$100.00
D	$\frac{1}{3}$	\$100.00
A	$\frac{1}{1}$	\$300.00
B	$\frac{1}{1}$	\$300.00

The chart has all my answers on it

The student models mathematics by using a table and diagrams to indicate the fields, their fractional value, worth, and geometric names. All labels and data are correct.

you have to break it into a fraction by seeing how many pieces it takes to make a hexagon. So 2 trapezoids make a hexagon and 2 rhombus and 2 triangles make a hexagon. Then divide \$1200.00 by 4

4 Hexagons and it is \$300.00 a hexagon

Trapezoid is $\frac{1}{2}$ - \$150.00

Rhombus is $\frac{1}{3}$ - \$100.00

Triangle is $\frac{1}{6}$ - \$50.00

2/150
3/100
6/300

My connection is to name each shape by the shape of the fields.

The student reasons correctly by naming each shape of the field.

The student uses precise mathematical language to support her/his viable argument. Some terms include the names of the shapes, fraction and money notation.

page 2 of 2

Grade 4 Math: Farmer Fred Annotated Student Work: Practitioner

Practitioner – Student 3 Summary

Achievement Level: Student 3 is a Practitioner according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Practitioner	The student's strategy of creating a table to show each field, determining the fractional part of each field, and the fair worth of each field, works to solve the problem. The student writes, "the chart has all my answers on it." The student's answer, "Field E and F is $\frac{1}{2}$ and \$150.00, Field G and H is $\frac{1}{6}$ and \$50.00, Field C and D is $\frac{1}{3}$ and \$100.00, Field A and B is $\frac{1}{1}$ and \$300.00," is correct.
Reasoning and Proof Practitioner	The student demonstrates correct reasoning of the underlying mathematical concepts in the problem. The student applies understanding of fractional parts of a whole when considering that all the fields combined total four hexagons. The student divides \$1200.00 by four for a quotient of \$300.00 and then divides the \$300.00 by the denominators two, three, and six to find the correct worth of each field.
Communication Practitioner	The student correctly uses the mathematical terms-fraction, worth, value, from the task. The student also correctly uses the terms-chart, hexagon, rhombus, triangles, trapezoid, shapes. The student correctly uses the mathematical notation- $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{1}$, \$150.00, \$50.00, \$100.00, \$300.00.
Connections Practitioner	The student makes the mathematically relevant connections, "My connection is to name each shape by the shape of the fields." The student diagrams a shape and labels it with its correct name.
Representation Practitioner	The student's table is appropriate and accurate with all columns correctly labeled and all entered data is correct. The student's diagram of the field shapes is also appropriate and accurate with each shape labeled correctly.

Instructional Implications: Farmer Fred, 4

Student Achievement Level: Practitioner

The following is a list of instructional implications that you may want to consider for students performing at the Practitioner level. In addition, you may want to consult the suggestions for the Novice and Apprentice levels:

- Include more writing of equations using fractions, $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6}$ which is one whole
- Encourage student to independently make more than one mathematically relevant connection about her/his work (see Preliminary Planning Sheet)
- Introduce another strategy to solve the same problem-area model/circle graph/diagram, table, number line,
- Establish a different total value for the fields for practice

Grade 4 Math: Farmer Fred Annotated Student Work: Apprentice

This student is an Apprentice according to both the Exemplars Rubric and the CCSS Content Standards Rubric (both included in the supporting materials).

Apprentice, Student 1

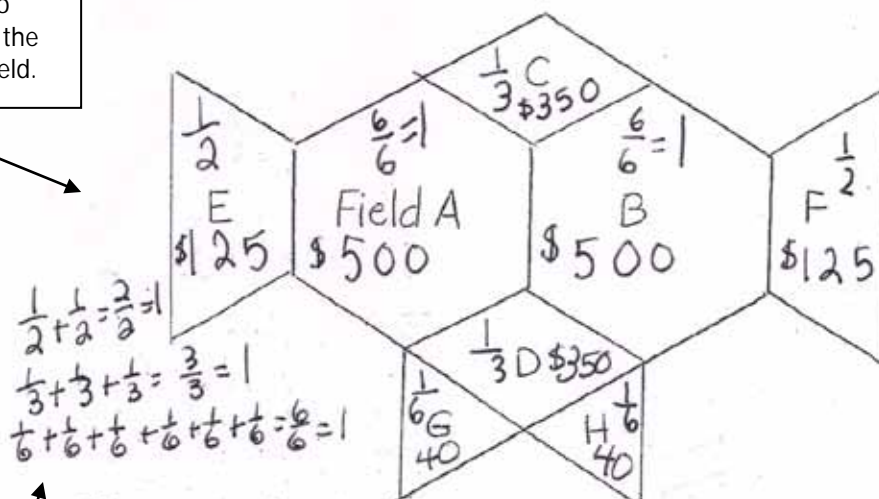
Grade 4

Name _____

Farmer Fred

The student is able to make sense of part of the problem. The student correctly defines the fractional value of each field but is not able to correctly define the worth of each field.

Farmer Fred's fields are worth twelve hundred dollars total. The fields are formed with the same properties as your pattern blocks. Each field's value is based on its size. What fraction of the total value is each field worth? How much is each field worth? Show and explain all of your mathematical thinking.



I need to find out the total value for each field and how much is each field worth. I will make a diagram and use my pattern blocks.

page 1 of 2

The student correctly extends understanding of fraction equivalence and ordering (4.NF 1, 2), and builds fractions from unit fractions by applying and extending previous understandings of operations on fractions (4.NF 3, a, b, d). The student does not use operations with whole numbers to solve the problem (4.OA 2).

Grade 4 Math: Farmer Fred
Annotated Student Work: Apprentice

hexagon = 500

Rhombus = 350

triangle = 40

Trapezoid = 125

Answer 1
The values
are $\frac{6}{6}, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}$

Answer 2
Each field is
worth 125,500
350 40

The student uses precise mathematical language to construct a viable argument. Some terms include the names of the shapes, total value, work, as well as fraction notation.

The student models with mathematics by diagramming some important features/shapes in the problem which are accurate and labeled correctly.

The student does not reflect on her/his solution to make connections to the regularities, structures, or trends in the problem.

page 2 of 2

Grade 4 Math: Farmer Fred Annotated Student Work: Apprentice

Apprentice – Student 1 Summary

Achievement Level: Student 1 is an Apprentice according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Apprentice	<p>The student's strategy of labeling each lot on the diagram of the eight fields with the correct fraction of the total value, works to solve the first part of the problem. The student's answer, "$\frac{6}{6}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$," is correct.</p> <p>The student's strategy of diagramming the four field shapes and stating their worth would work to solve the second part of the problem but the student does not determine any correct values. The student's answer, "Each Field is worth 125, 500, 350, 40," is incorrect.</p>
Reasoning and Proof Apprentice	<p>The student demonstrates correct reasoning of some of underlying mathematical concepts in the problem. The student applies understanding of fractional parts of a whole when considering hexagon, trapezoid, rhombus, and triangle shapes and determines what fraction of the total value each field is worth. The student is not able to determine any correct worth of a field. The student does not supply any computation to support her/his thinking so one does not know why the student uses 500, 350, 40, 125. The money the student lists does not connect with the student's understanding of fractional parts of a whole.</p>
Communication Practitioner	<p>The student correctly uses the mathematical terms-total value, worth, from the task. The student also correctly uses the terms-diagram, hexagon, rhombus, triangle, trapezoid.</p> <p>The student correctly uses the mathematical notation-$\frac{1}{2}$, $\frac{6}{6}$, $\frac{1}{3}$, $\frac{1}{6}$.</p>
Connections Novice	<p>The student solves the problem and stops without making a mathematically relevant connection.</p>
Representation Apprentice	<p>The student's diagram of the shapes from the problem is appropriate but not accurate. The student correctly diagrams and labels each shape but the worth of each field is incorrect. No money notation was used to support that the numbers were actually what each field was worth.</p>

Grade 4 Math: Farmer Fred Annotated Student Work: Apprentice

This student is an Apprentice according to both the Exemplars Rubric and the CCSS Content Standards Rubric (both included in the supporting materials).

Apprentice, Student 2

Grade 4

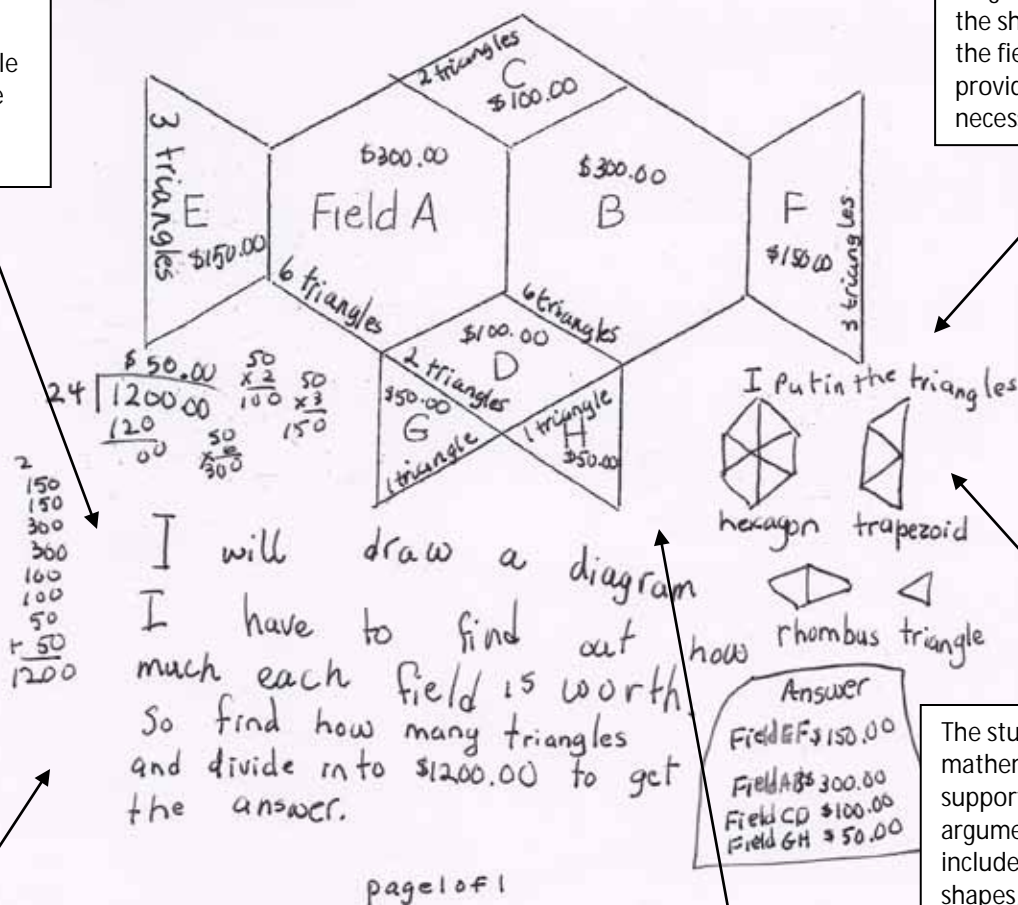
Name _____

Farmer Fred

Farmer Fred's fields are worth twelve hundred dollars total. The fields are formed with the same properties as your pattern blocks. Each field's value is based on its size. What fraction of the total value is each field worth? How much is each field worth? Show and explain all of your mathematical thinking.

The student correctly extends understanding of fraction equivalence and ordering (4.NF 1, 2), but does not build fractions from unit fractions by applying and extending previous understandings of operations on fractions (4.NF 3 a, b, d). The student uses operations with whole numbers to solve the second part of the problem (4.OA 2).

The student models with mathematics by accurately diagramming the shapes of the fields and providing all necessary labels.



The student does not reflect on her/his solution to make a connection about the structures, regularities, or trends in the problem.

The student is able to make sense of most of the problem. The student applies fractional understanding by determining the correct number of triangles for each shape but does not label the fractional value of each field the triangles represent. The student finds the correct worth of each field.

Grade 4 Math: Farmer Fred

Annotated Student Work: Apprentice

Apprentice – Student 2 Summary

Achievement Level: Student 2 is an Apprentice according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Apprentice	The student's strategy of diagramming the fractional parts of each shape by indicating how many triangles make up each shape would work to solve the first part of the problem but the student does not state any fractions as an answer. The student shows computation to support how each field's worth was determined. The student's answer, "Field E, F \$150.00. Field A, B \$300.00, Field C, D \$100.00, Field G, H \$50.00," is correct.
Reasoning and Proof Apprentice	The student demonstrates correct reasoning of most of the underlying mathematical concepts in the problem. The student applies understanding of fractional parts of a whole when considering how many triangles are needed to create a hexagon, trapezoid, and rhombus. The student writes how many triangles are needed in each shape but does not represent that thinking with a fraction for each field which is why the student earns a performance level of Apprentice. The student correctly reasons and labels the worth of each field and includes computation to support her/his thinking.
Communication Practitioner	The student correctly uses the mathematical term—worth—from the task. The student also correctly uses the terms-diagram, hexagon, trapezoid, rhombus, triangle. The student correctly uses the mathematical notation-\$150.00, \$300.00, \$100.00, \$50.00.
Connections Novice	The student solves the problem and stops without making a mathematically relevant connection.
Representation Apprentice	The student's diagram of the shapes from the problem with the triangles in each shape is appropriate and accurate. The shapes are correctly labeled and the student states, "I put in the triangles."

Grade 4 Math: Farmer Fred Annotated Student Work: Apprentice

Exemplars Rubric: Apprentice
CCLS Content Rubric: Practitioner

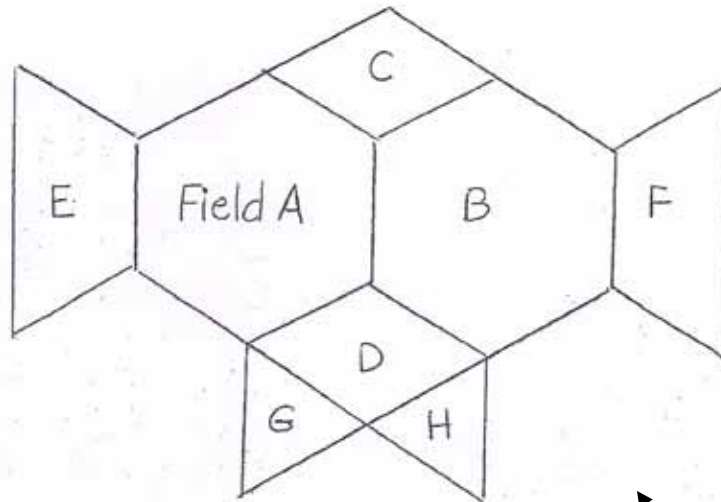
This student is an Apprentice according to the Exemplars Rubric and a Practitioner according to the CCLS Content Standards Rubric (both included in the supporting materials).

Grade 4


Name _____

Farmer Fred

Farmer Fred's fields are worth twelve hundred dollars total. The fields are formed with the same properties as your pattern blocks. Each field's value is based on its size. What fraction of the total value is each field worth? How much is each field worth? Show and explain all of your mathematical thinking.



I have to find out how the fraction and the total value is of each field is. I will use a diagram to see how many there are.



hexagons

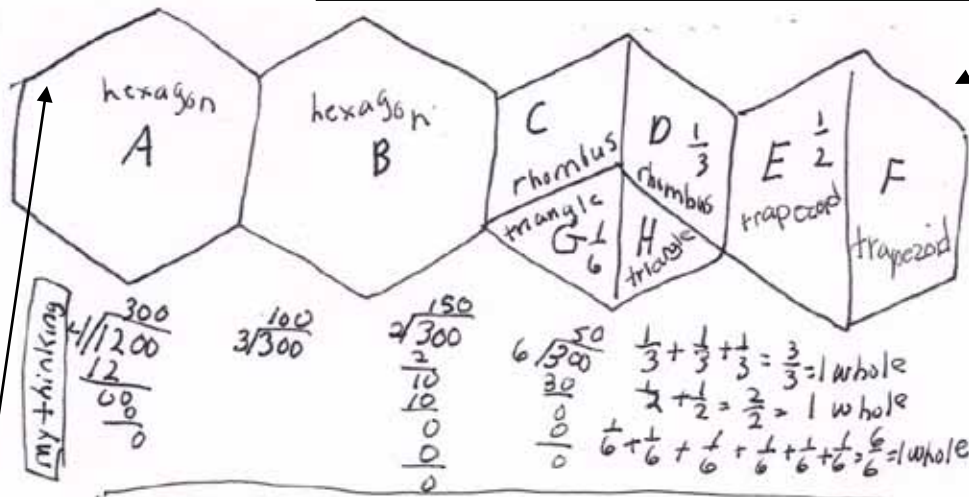
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The student is able to make sense and persevere in solving the problem. The student applies understanding of fractional parts of a whole in determining the correct fractional value of each field and applies correct number and operations thinking to find the correct worth of each field.

Grade 4 Math: Farmer Fred

Annot

The student correctly extends understanding of fraction equivalence and ordering (4. NF 1, 2), builds fractions from unit fractions by applying and extending previous understandings of operations on fractions (4.NF 3 a, b, d), and uses operations with whole numbers to solve the second part of the problem (4.OA 2).



Answer 1: Field A = 1 whole Field B = 1 whole
 Field C = $\frac{1}{3}$ Field D = $\frac{1}{3}$
 Field E = $\frac{1}{2}$ Field F = $\frac{1}{2}$
 Field G = $\frac{1}{6}$ Field H = $\frac{1}{6}$

Answer 2: Field A = \$300 Field B = \$300.00
 Field C = \$100 Field D = \$100
 Field E = \$150 Field F = \$150
 Field G = \$50 Field H = \$50.00.

The student does not reflect on her/his solution to make a connection about the structures, regularities or trends in the problem.

The student uses precise mathematical language to support her/his viable argument. Some terms include the names of the shapes, fraction and money notation.

The student models with mathematics by accurately diagramming the shapes of the fields and providing all necessary labels.

page 2 of 2

Grade 4 Math: Farmer Fred Annotated Student Work: Apprentice

Apprentice/Practitioner – Student 3 Summary

Achievement Level: Student 3 is an Apprentice according to the Exemplars Process Rubric and a Practitioner according to the CCSS Content Standards Rubric (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Practitioner	The student's strategy of determining and diagramming four hexagons that can be formed using all the fields, dividing 1200 by 4, 300 by 3, 300 by 2, 300 by 6, stating the quotients as money, and finding the fraction of the total value each field is, works to solve the problem. The student's answer for part one of the problem, "Field A = 1 whole, Field B = 1 whole, Field C = $\frac{1}{3}$, Field D = $\frac{1}{3}$, Field E = $\frac{1}{2}$, Field F = $\frac{1}{2}$, Field G = $\frac{1}{6}$, Field H = $\frac{1}{6}$," is correct. The student's answer for part two of the problem, field A = \$300 Field B = \$300.00, Field C = \$100, Field D = \$100, Field E = \$150, Field F = \$150, Field G = \$50, Field H = \$50.00, is correct.
Reasoning and Proof Practitioner	The student demonstrates correct reasoning of the underlying mathematical concepts in the problem. The student applies understanding of fractional parts of a whole when considering hexagon, trapezoid, rhombus, and triangle shapes and determines what fraction of the total value each field is worth. The student also shows correct reasoning in determining the worth of each field by dividing 1200 by the 4 hexagons and then using the denominator of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$ to find the correct worth of each field.
Communication Practitioner	The student correctly uses the mathematical terms-fraction, total value, from the task. The student also correctly uses the terms-diagram, hexagon, rhombus, triangle, trapezoid, whole. The student correctly uses the mathematical notation- $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{3}$, $\frac{2}{2}$, $\frac{6}{6}$, \$300.00, \$50.00. The student does not earn notation credit for \$300, \$100, \$150, \$50, because they lack a decimal point and the zeros to hold the cent places.
Connections Novice	The student solves the problem and stops without making a mathematically relevant connection.
Representation Practitioner	The student's diagram is appropriate and accurate with all necessary labels included.

**Grade 4 Math: Farmer Fred
Annotated Student Work**

Instructional Implications: Farmer Fred, 4

Student Achievement Levels: Novice and Apprentice

The following is a list of instructional implications that you may want to consider for students performing at the Novice and Apprentice levels:

Review how to read a mathematics problem-listen to the problem being read, read the problem to yourself, underline the important information, find the question sentence(s), determine the important nouns, look for mathematical language, etc.

Explore congruency and equivalent fractions using pattern blocks

- Use other manipulatives to investigate fractional part of a whole-fraction bars and circles, fraction wheels, paper plates, sticks
- Use games requiring the use of fractions with and without like denominators
- Work with a number line
- Order fractions with cards, sticks, etc.
- Provide activities where student adds and subtracts fractions with like and unlike denominators
- Review mathematical language-model, diagram/area model/circle graph key, table, number line, more/less than, fraction, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{6}$, $\frac{6}{6}$..., numerator, denominator, whole, equivalent, shapes, hexagon, trapezoid triangle, rhombus, polygon, congruent
- Review how to make a diagram/area model with a key, table, chart
- Have centers available for investigation and practice with finding and applying fractions
- Provide leading questions to begin reflection on the solution in order to see regularities, structures, patterns, trends, etc. (See Preliminary Planning Sheet)
- Review division and multiplication

Grade 4 Math: Farmer Fred Annotated Student Work: Novice

This student is a Novice according to both the Exemplars Rubric and the CCSS Content Standards Rubric (both included in the supporting materials).

The student is not able to discern that the problem requires an understanding of fractional parts to a whole and computation with money. The student does not indicate a fractional value for each field and the worth of each field is not correct.

Novice, Student 1

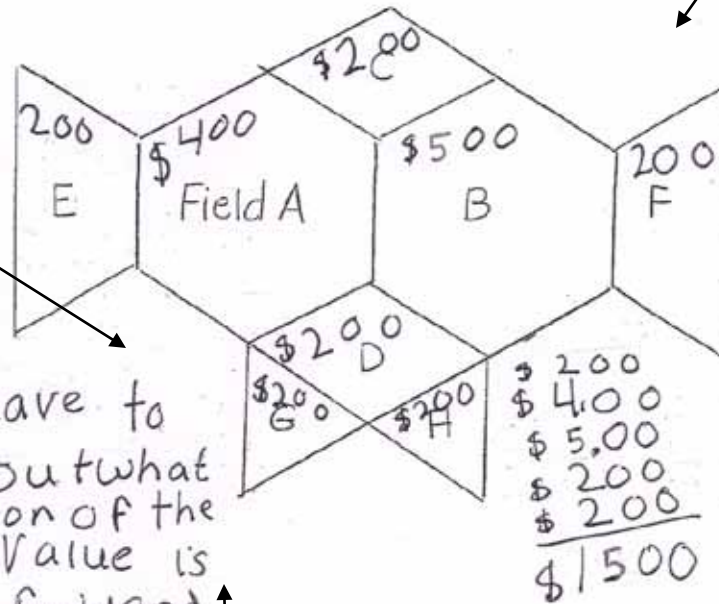
Grade 4

Name _____

Farmer Fred

Farmer Fred's fields are worth twelve hundred dollars total. The fields are formed with the same properties as your pattern blocks. Each field's value is based on its size. What fraction of the total value is each field worth? How much is each field worth? Show and explain all of your mathematical thinking.

The student does not apply understanding of fraction equivalence and ordering (4NF 1, 2), and does not build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers (4NF 3, b, d).



I have to find out what fraction of the total value is each field and how much is each field

\$200
\$400
\$500
\$200
\$200
\$1500

page 1 of 1

The student applies the math term, total value, in the solution but not the stated word-fraction.

The student does not model with mathematics. The student only labels the one provided with incorrect monetary amounts for each field and does not construct her/his own diagram or table to construct a viable argument.

The student is not able to search for the regularity and trends embedded in the problem.

Grade 4 Math: Farmer Fred Annotated Student Work: Novice

Novice – Student 1 Summary

Achievement Level: Student 1 is a Novice according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Novice	The student's strategy of filling in each lot on the diagram of the eight fields with either \$200, \$400, \$500, or 200 would not work to solve the problem. The student does not state an answer.
Reasoning and Proof Novice	The student does not demonstrate correct reasoning of the underlying mathematical concepts in the problem. The student does not apply understanding of fractional parts of a whole when considering hexagon, trapezoid, rhombus, and triangle shapes, equivalent fractions, and correct money notation. The student is not able to determine any correct worth of a field and labels fields with the same properties and fractional parts to the whole differently or with the same worth given another field's shape. The student does not address the fraction of the total value each field is worth.
Communication Apprentice	The student does not use any mathematical language.
Connections Novice	The student solves the problem and stops without making a mathematically relevant connection.
Representation Novice	The student does not attempt a mathematical representation. Filling in the fields on the diagram provided is not considered an attempt. The student has to make her/his own mathematical representation.



GRADE 4 MATH: FARMER FRED

INSTRUCTIONAL SUPPORTS

The instructional supports on the following pages include a unit outline with formative assessments and suggested learning activities. Teachers may use this unit outline as it is described, integrate parts of it into a currently existing curriculum unit, or use it as a model or checklist for a currently existing unit on a different topic.

Unit Outline – Grade 4 Math

INTRODUCTION: This unit outline provides an example of how to integrate performance tasks into a unit of instruction. *Teachers may (a) use this unit as it is described below; (b) integrate parts of it into a currently existing curriculum unit; or (c) use it as a model or checklist for a currently existing unit on a different topic*

Grade Subject: Title

UNIT TOPIC AND LENGTH:

- Understanding how shapes and sizes fit together. Can be segmented into parts to help learners make sense of problems and persevere in solving them by using their knowledge of not only lines and angles, but also the four operations with whole numbers. Further knowledge of fractions and how they can be applied to geometry is fundamental to recognizing equalities in shape and area.
- Unit Length: 4-6 weeks

COMMON CORE CONTENT STANDARDS:

- **3.G.2** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.
- **4.G.2** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles
- **4.NF3c** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
- **4.NF3d** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
- **4.NF.4c** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.
- **4.OA.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
- **MP.1** Make sense of problems and persevere in solving them.
- **MP.3** Construct viable arguments and critique the reasoning of others.
- **MP.6** Attend to precision.
- **MP.7** Look for and make use of structure.

BIG IDEAS/ENDURING UNDERSTANDINGS:

ESSENTIAL QUESTIONS:

Unit Outline – Grade 4 Math

<ul style="list-style-type: none"> ➤ Mathematicians recognize equalities in shape and area, connect geometry to number, operations, and measurement via the notion of partitioning. ➤ Mathematicians classify shapes by properties of their lines and angles and make mathematical sense of shapes based on their geometric properties. ➤ Mathematicians make sense of problems by persevering, applying, and modeling, to expand the concept of number sense beyond whole numbers, to include fractions and irrational numbers. 	<ul style="list-style-type: none"> ➤ How can I combine shapes to find the area of the new shapes? ➤ How can I group shapes to identify their similarities? ➤ What steps do I need to take to make sense of problems?
<p>CONTENT:</p> <p>Connections</p> <ul style="list-style-type: none"> ➤ Geometry to numbers ➤ Operations ➤ Equalities of shape and area ➤ Measurements ➤ Parts to whole <p>-----</p> <p>Classification</p> <ul style="list-style-type: none"> ➤ Properties of lines and angles ➤ Geometric properties ➤ Characteristics of polygons ➤ Proportional relationships <p>-----</p> <p>Number Sense</p> <ul style="list-style-type: none"> ➤ Fractions ➤ Whole numbers ➤ Decimals ➤ Money notation ➤ Irrational numbers 	<p>SKILLS:</p> <ul style="list-style-type: none"> ➤ Partition shapes into parts with equal areas. ➤ Combine polygons to determine the area of irregular shapes ➤ Construct models with multiple representations of the same value ➤ Identify appropriate operations within the problem solving process <p>-----</p> <ul style="list-style-type: none"> ➤ Describe the characteristics of polygons. ➤ Compare the characteristics of polygons. ➤ Identify the similarities in polygons. <p>-----</p> <ul style="list-style-type: none"> ➤ Add fractions, decimals, and whole numbers. ➤ Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. ➤ Formulate explanations of processes taken to problem solve. ➤ Apply concepts to solve word problems involving multiplication of a fraction by a whole number ➤ Write decimal numbers in money notation.

Unit Outline – Grade 4 Math

ASSESSMENT EVIDENCE AND ACTIVITIES:

INITIAL ASSESSMENT: 75 POINTS

The **initial assessment** also allows for what is sometimes called a touchstone task. The task should be rich enough that it can be solved from a variety of approaches, so that students can make sense of it in natural ways. Then as the unit progresses, students should be able to move to more efficient or grade-level appropriate strategies. As the students learn new ideas or procedures, students and the teacher can reflect upon how these new ideas and procedures might apply to the initial task.

The task 75 Points asks students to partition shapes into parts with equal areas, find the fair value of each shape by multiplying fractions by whole numbers and find the total value of all shapes through addition of mixed numbers. Students must justify their mathematical thinking as they solve the problem. *See 75 Points for full details.*

FORMATIVE ASSESSMENTS:

The purpose of formative assessment is to surface misconceptions and, through the course of the lessons, to provide ways for students to resolve these misconceptions and deepen their understanding. By surfacing misconceptions, the teacher is then able to make mid-unit corrections to instruction. Thus, students' experiences help to improve learning, rather than waiting until the final assessment to uncover problems or gaps in learning. Throughout this unit, periodic collection and analysis of work from the tasks included in the learning plan should yield a wealth of information teachers can use formatively.

FINAL PERFORMANCE TASK: FARMER FRED

This task gives students the total monetary value of a set of fields and asks them to find the fractional and actual value of each field. Students use fractional parts of a whole, properties of shapes, congruency, and computation using fractions and money to determine the fair value and total worth of eight fields. Students must justify their mathematical thinking as they solve the problem. *See Farmer Fred for full details.*

LEARNING PLAN & ACTIVITIES:

PATTERN BLOCKS, FRACTIONS AND TANGRAMS:

- In addition to the supports below, teachers may use some of the tasks included in this packet as part of the learning plan throughout the unit. A suggested sequence for use of the tasks is below:
 - Little Bear
 - Kickball Money
 - Golden Pin
 - Selling Fudge
 - Build it with Pattern Blocks
 - A Pattern Block Design

Unit Outline – Grade 4 Math

- A Challenge #2
- Feverish Freddy
- Build a Shape Workshop: Students will work to replicate a shape (i.e.: the yellow hexagon or the blue trapezoid) using a different assortment of pattern blocks. Next, ask students to find multiple ways to do this. As they develop combinations, students should record them using fractions with the original shape (the yellow hexagon or blue trapezoid) having a value of 1. For example, if they build the hexagon with one red trapezoid and three green triangles, they'll write: $1/2 + 1/6 + 1/6 + 1/6 = 1$.
- Ask students to investigate which of the pattern blocks they can use to build shapes that are larger but similar--such as four or nine squares to make a larger square. To get children started, ask: Can you use green triangles to build a larger green triangle that is still the same shape? How many do you need?
- Edible Fraction Activities
Begin by reading the book *Eating Fractions* or another appropriate book to introduce the concept of fractions as being a part of a whole or a set. Use the graham crackers to illustrate wholes, halves and quarters, the Hershey Bars to explore sixths and eighths, and the colored marshmallows to determine the sets of colors. To introduce denominators and numerators ask students to eat $1/2$, $2/6$, etc. of the food.
- The Tangram puzzle was invented by the ancient Chinese hundreds of years ago. It is a square broken into seven pieces. When rearranged, these pieces form a great variety of shapes and pictures. This puzzle provides an excellent background for determining fractional parts and wholes.

Ask students to create a square unit from all seven pieces.

- Choosing each piece of the Tangram set, ask students which part of the whole square it is.
 - Large right triangles: $1/4$
 - Square: $1/8$
 - Small right triangles: $1/16$ each
 - Medium Right Triangle: $1/8$
 - Parallelogram: $1/8$
- Changing the unit whole is an interesting activity. For example, if a large right triangle is chosen and announced to be 1 unit in area, ask how the other pieces correspond.
 - Square: $1/2$
 - Small Right Triangle: $1/4$ (each)
 - Parallelogram: $1/2$
 - Medium Right Triangle: $1/2$

RESOURCES:

- <http://www.mathplayground.com/patternblocks.html>
- <http://www2.scholastic.com/browse/article.jsp?id=4353>
- <http://math.rice.edu/~lanius/Patterns/>

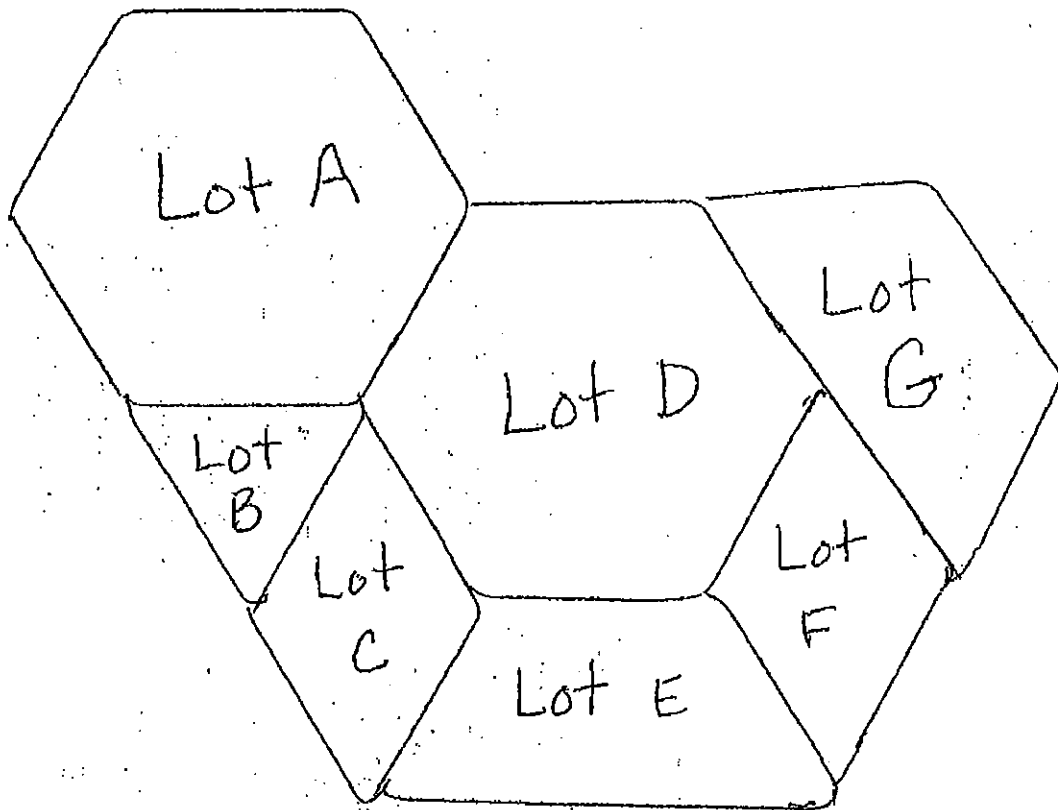
Unit Outline – Grade 4 Math

- Eating Fractions, Bruce Mcmillan
- <http://mathman.biz/html/prob2.html>
- Plastic Pattern Blocks (set of 250)
- Pattern cards for pattern blocks set activity cards
- 5X5 Pin Geo-boards
- Greedy Triangle, Marilyn Burns
- Mummy Math: An Adventure in Geometry, Cindy Nueschwander

Name _____

75 points

Mike and Juan are walking to the lots in the diagram below. The lots are formed with the same properties as your pattern blocks. They are going to clean each lot by removing papers, bottles, and cans. Each lot they clean will earn Mike and Juan points. If they earn a total of at least 75 points they will earn a reward. If Lot D is worth 20 points to clean up, what is the fair value of the other lots? If Mike and Juan clean up all the lots will they have earned enough points to earn a reward? Show all your mathematical thinking.



P/S	R/P	Com	Con	Rep	AC/LV

Name: _____

Little Bear

Little Bear wants to make a symmetrical design using yellow, red, green, and blue pattern blocks to put on his vest. The value of each green block is ten cents. Each colored block's cost is proportional to the triangle's cost. Design a pattern for Little Bear's vest that is worth four dollars. Use at least two of each of the shapes you choose when you make your design. Show all your mathematical thinking.

Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task (4)

Number + Operations - Fractions

Title of Task Little Bear Content Strand(s) Addressed Geometry

Program Link

State Standard(s) Addressed

Common Core Standard(s) 4.NF.a.3.a.d 4.G.3

Underlying Mathematical

Concepts

Symmetry
Properties of hexagon, trapezoid, triangle, rhombus
money notation
fair value
multiplication/division
number sense to \$4.00

Problem Solving

Strategies/Representation

Model (manipulatives)

diagram
key
chart
guess and check

Mathematical Language

Model
diagram
key
chart
symmetry
triangle
hexagon
trapezoid
rhombus
money notation
\$ value
shape

polygon
total
per
double
congruent
flip
rotate
similar
area
square
fractions $\frac{1}{2}$ $\frac{2}{3}$
decimals .50 $\frac{1}{2}$
percent 50%

Possible Solution(s)

Possible Design Answer

A Design

Shape	Total Money
hexagon	\$1.20
trapezoid	\$1.20
rhombus	.40
triangle	\$1.20

1 whole 100%
 $.50 \frac{1}{2}$ 50%
 $.33\frac{1}{3}$ $\frac{1}{3}$...
 $\frac{1}{6}$...

\$60. 330×20 \$10
 $\times 4 \times 2$
 $\times 12$
 $\$120$ $\$120 \times 40$ \$1.20 \$4.00
 $\$1.20 + \$1.20 + \$1.20 + \1.20 \$4.00

Connections

- name the shapes
- create a new design
- recreate design with triangle having a new value using divisibility rules
- relate to a similar problem and state math
- Discuss why square pattern block can not be used
- Design a shape with more than 2 lines of symmetry
- rhombus value is double A
- find fractional value or percent/decimal of each shape

Related Tasks

See Resource Binder

P/S	R/P	Com	Con	Rep	AC/LV

Name _____

Kickball Money

Mr. Barton's class decided to sell cookies to earn enough money to buy a kickball for \$8.00. The class baked cookies in the shape of the pattern blocks like the ones in the classroom (except for the rhombus and the square). The class decided to sell each hexagon shaped cookie for twenty-four cents. What would be a fair price to charge for the other two shaped cookies? The class sold all the different shaped cookies and earned exactly \$8.00. How many of each cookie could the class have baked? Show all your mathematical thinking.

Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task

④

Number Operations - Fractions

Title of Task Kickball Money

Content Strand(s) Addressed Geometry

State Standard(s) Addressed

Program Link

Common Core Standard(s) 4.NF.A.3.a-d 4.G.3

Underlying Mathematical

Problem Solving

Concepts

Strategies/Representation

Fractional part of a whole

Diagram (Key)

properties of hexagon, triangle, trapezoid,

model (manipulatives)

rhombus, square

chart

congruency

guess and check

fair value

multiplication / addition / division

money notation

Mathematical Language

Diagram

hexagon

key

congruent

model

fair value

chart

money notation

properties

\$.

shape

area

Combinations most/least

equivalent

fraction

triangle

$\frac{1}{2}, \frac{1}{6}, \frac{2}{6}$

rhombus

percent 50%

square

line of symmetry

Related Tasks decimals: 50%




Set Resource Binder

Possible Solution(s)

Answer

fair price

shape Name of shape Value fractional part

	hexagon	\$.24	1 whole
	trapezoid	\$.12	$\frac{1}{2}$
	triangle	\$.04	$\frac{1}{6}$

shape	Name of shape	Value	fractional part
\$.24	hexagon	\$.24	1 whole
\$.12	trapezoid	\$.12	$\frac{1}{2}$
\$.04	triangle	\$.04	$\frac{1}{6}$
\$.40			

Number of cookies	Shape / Money earned
20	hexagon \$4.80
20	trapezoid \$2.40
20	triangle \$0.80

\$4.80
\$2.40
\$.80
\$8.00

Connections

Can't use a square as it is not congruent

Names trapezoid and triangle

Show other totals for \$8.00

Show fractional / decimal

Equivalent names

Show percents $\frac{1}{2}$ is 50%

\$.12 is one-half of \$.24 so

can have 10 hexagon and

40 trapezoid cookies

Relate to a similar problem

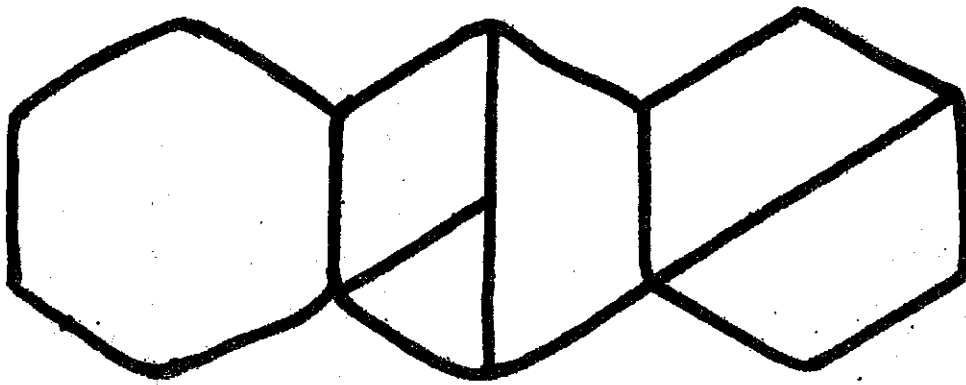
and state math

Name _____

P/S	R/P	Com	Con	Rep	AC/LV

The Golden Pin

The pin below has been passed down through many generations. The pin is divided into three large parts that look like the yellow pattern block. The first part is worth sixty dollars. Based on this information, what is the fair value of each section of the second part and each second of the third part of the golden pin? Show all your mathematical thinking.



Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task

(4)

Number + Operations - Fractions

Content Strand(s) Addressed

Geometry

Title of Task The Golden Pin

State Standard(s) Addressed

Common Core Standard(s) 4.NF.2, 3, 4, 4.G.3

Underlying Mathematical

Concepts

Congruency
fractional part of a whole
properties of hexagon, rhombus, triangle,
multiplication (addition/division)
money notation
number sense to \$60.00

Problem Solving

Strategies/Representation
Model (manipulatives)
diagram key
chart

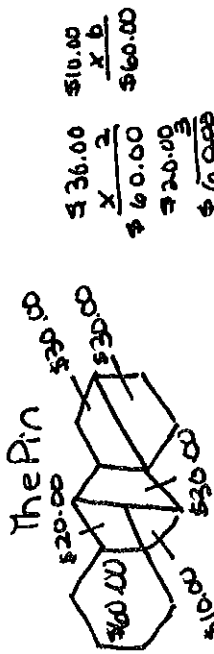
Mathematical Language

model
diagram
key
chart
shape
hexagon
rhombus
triangle
trapezoid
square
fractions
Related Tasks combinations

per
total
product
sum
congruent
money notation
\$. 4.10 of symmetry
 $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$
whole
area
fair value
decimals

Sec Resource Binder

Possible Solution(s)



Shape/Name	Fair Value	Fractional Part
hexagon	\$60.00	1/whole
rhombus	\$20.00	$\frac{1}{3}$
triangle	\$10.00	$\frac{1}{6}$
trapezoid	\$30.00	$\frac{1}{2}$

Connections

- names each shape correctly
- finds fractional and decimal equivalent value
- hexagon has most area
- triangle has least area
- can not use square pattern block - not congruent
- Total value of pin is \$180.00
- Redesign pin - could have 3 hexagons or 6 trapezoids or 9 rhombi or 18 triangles or combinations of each
- Relate to a similar problem and state math

Answer
see diagram

There are lines of symmetry in the 2nd + 3rd hexagons

P/S	R/P	Com	Con	Rep	AC/LV

Name _____

Selling Fudge

Some students make fudge for a bake sale. The students cut the fudge in the same sizes as the yellow, green, red, and blue pattern block shapes they use in math class. The students sell the fudge cut like a green pattern block for five cents. What is the fair price the students charge for the other shapes of fudge?

Ben wants to spend fifty cents to buy fudge. What pieces of fudge can Ben buy?

Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task

(4)

Title of Task Selling Fudge Content Strand(s) Addressed Number + Operations - Fractions

State Standard(s) Addressed Common Core Standard(s) 4.NF.A.3.a-d 4.G.3 Program Link

Underlying Mathematical

Concepts

fractional part of a whole

congruency

money notation \$.

properties of hexagon, trapezoid,

triangle, rhombus

multiplication / division

Problem Solving

Strategies/Representation

model (manipulatives)

diagram / key

chart

Mathematical Language

model

diagram

key

chart

hexagon

trapezoid

rhombus

triangle

square

area

cost/profit

total

Related Tasks

money notation

\$.

most

least

congruent

per

equivalent

vertex

combinations

numerator

angle

denominator

side

fractions $\frac{1}{2}$...





decimals .50...

percents 50%...

Possible Solution(s)

Fudge

Answer

shape	shape name	value	fractional value
	triangle	\$0.05	$\frac{1}{20}$
	rhombus	\$0.10	$\frac{1}{10}$
	trapezoid	\$0.15	$\frac{1}{6}$
	hexagon	\$0.30	1 whole

Answer 2 - Various combinations

for \$.50

example $\square + \square + \triangle$

$\$0.30 + \$0.15 + \$0.05 = \0.50

Connections

. State names of each shape

. Add fractional, percent, or decimal amount

. hexagon is most expensive

. triangle is least expensive

. Assign a new value use the

divisibility rules

. Show more than one way

to spend \$.50

. Compare area and congruency

. Relate to a similar problem

and state math

. Explain why the square pattern

block can't be used

See Resource Binder

P/S	R/P	Com	Con	Rep	AC/LV

Name _____

Build it With Pattern Blocks

Jon is building a design with pattern blocks. The yellow hexagon is worth eighteen cents. Jon will create a design that is worth one dollar and fifty-six cents. Jon will use fifteen or more pattern blocks. Jon will not use any square pattern blocks. What could Jon's design look like? How does Jon know that the design is worth one dollar and fifty-six cents? What is the greatest number of blocks that can be used to create a design worth one dollar and fifty-six cents? Show all your mathematical thinking.

Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task

④

Number Operations - Fractions
Geometry

Title of Task Build it With Pattern Blocks

Content Strand(s) Addressed

Program Link

State Standard(s) Addressed

Common Core Standard(s) 4.NF.2, 3a, d 4.G.3

Underlying Mathematical

Concepts

Fractional part of a whole
properties of shapes
number sense to \$1.56
money notation
multiplication / addition
comparison

Problem Solving

Strategies/Representations

Model (manipulatives)

Diagram / Key
table

Mathematical Language

model value
diagram per
key fractions
table $\frac{1}{2}, \frac{1}{3}, \dots$
triangle decimals
hexagon .50...
trapezoid percents
square 50%...
rhombus shape
money notation most/least
\$ symmetrical
double

Related Tasks

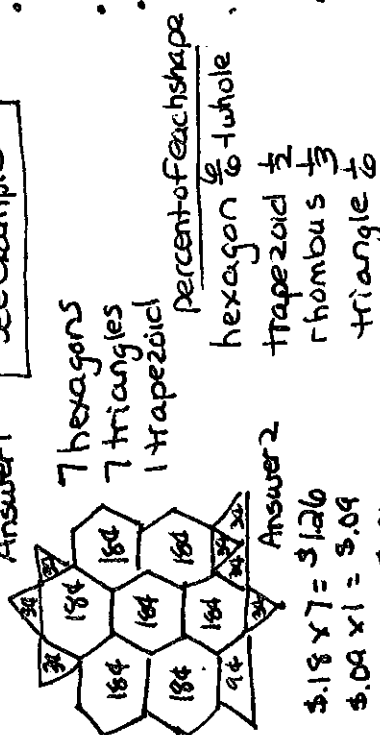
Connections
A rhombus would value double
a triangle

- The design is symmetrical
- give the percent value of each shape
- give the decimal value of each shape
- give fraction for value of each shape
- Relate to a similar problem and state math
- Make additional designs that could total \$1.56

Possible Solution(s)

Answer 1

Answer
number of
solutions for 1, 2
see example

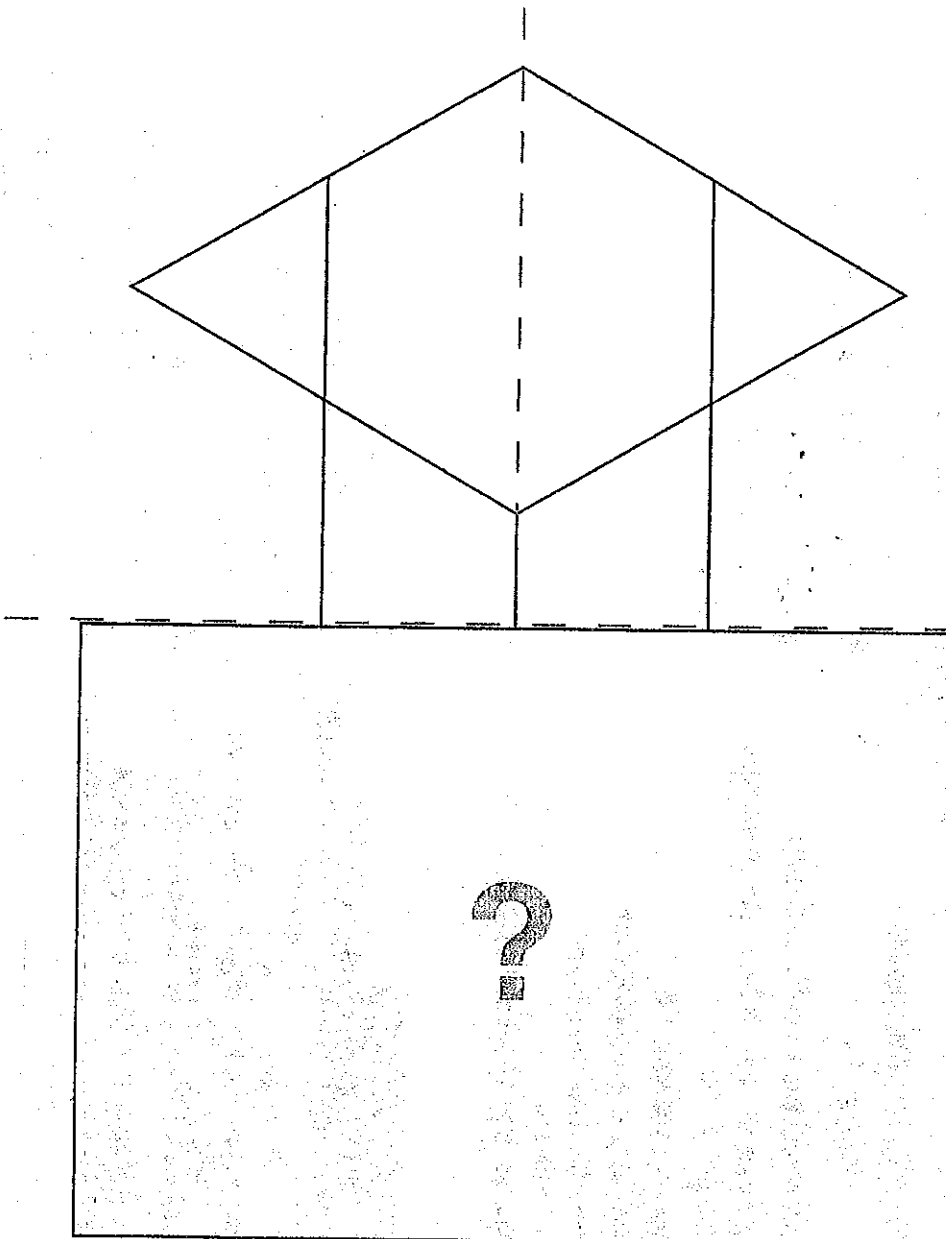


P/S	R/P	Com	Con	Rep	ACLV

Name _____

A Pattern Block Design

Tom made a design with pattern blocks. Then Tom drew in two lines of symmetry. Tom covered part of the design with a piece of paper. Tom asked Jen to discover what was under the paper. Tom also told Jen that the hexagon was worth twelve dollars and asked her to determine the fair value of the other shapes in the design. How did Jen complete Tom's design and what values did Jen give the pattern block shapes? Show all your mathematical thinking.



Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task ④

Number Operations - Fractions
Geometry

Title of Task A Pattern Back Design

State Standard(s) Addressed _____

Content Strand(s) Addressed _____

Program Link _____

Common Core Standards) 4.NF.A.3.a-d 4.G.3

Underlying Mathematical

Problem Solving

Concepts

Strategies/Representation

Mathematical Language

fractional part of a whole
properties of triangle, hexagon
trapezoid

Model (Manipulatives)
diagram (key
chart

model
diagram
key
chart
money
money notation
fair value
\$.
line of symmetry
congruent
per
triangle
hexagon
trapezoid
rhombus

fractions
1/2, 2/3... whole
decimals .30...
percent 100%...
total
fair value
shapes
congruent
area
flip
rotate
numerator
denominator
Related Tasks




money notation \$.
multiplication / addition
division / subtraction
fair value

number sense to \$12.00

57

Possible Solution(s)

Answer - finish pattern by adding
2 trapezoids, 1 hexagon, 2 triangles

Design	shape	name of shape	fractional value
	hexagon	\$12.00	1 whole
	trapezoid	\$6.00	1/2
	triangle	\$2.00	1/6

$$\frac{9}{2} \sqrt{12}$$

$$\frac{2}{6} \sqrt{12}$$

Connections

total for all shapes is \$20.00

total value of design is

\$44.00

compare area

show fractional decimal / percent

value of each shape

prove why a square can not be
used as it is not congruent

create a new design

assign a different value for
the hexagon.

Discuss how some hexagon
value would not work "cents" issue

Relate to similar
problem and state
math

See Resource Binder

P/S	R/P	Com	Con	Rep	AC/LV

Name _____

A Challenge #2

Emma thinks of a mathematical challenge for the students in the class to solve. Emma gives the students the following directions:

1. Draw a square on the paper.
2. Draw in the lines of symmetry.

Emma asks the following question: If the square is worth seventy-four dollars, what are the fair values of at least three shapes in your square? Show all your mathematical thinking.

Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task ④

Title of Task A Challenge #2 Content Strand(s) Addressed Number + Operations - Fractions

State Standard(s) Addressed Common Core Standard(s) 4.NF.A.3.a-d 4.G.3 Program Link

Underlying Mathematical

Concepts

Number sense to \$74.00
fractional part of a whole
properties of square, rectangle,
triangle
congruency / fair value
symmetry
division / multiplication

Problem Solving

Strategies/Representation
model (manipulatives)
diagram / key
chart

Mathematical Language

model
diagram
key
chart
square
rectangle
triangle
trapezoid
polygon
symmetry
fractions $\frac{1}{2}, \frac{1}{4}, \dots$
percents 50%...
decimals .50...
money notation
\$. $\frac{1}{2}$ $\frac{1}{4}$
equal
total
flip, rotate
area
fair value
similar
congruent

Possible Solution(s)

Answer 1
Square
Answer 2
3 of following
small Δ \$9.25
med. Δ \$18.50
large Δ \$37.00
small \square \$18.50
rectangle \$37.00
rt. trapezoid \$27.75



shape / name of shape	value in money	value in %	value in decimal

A possible chart to show

$$\begin{array}{r}
 \$9.25 \times 4 = \$37.00 \\
 \$18.50 \times 2 = \$37.00 \\
 \$27.75 \times 1 = \$27.75 \\
 \hline
 \text{Total} = \$101.75
 \end{array}$$

Connections

- Finds more than 3 shape values
- compare to a rectangle, trapezoid, etc.
- Discuss how areas compare
- Show fraction, decimal, percent value of shapes
- name the shapes
- Relate to a similar problem and state math
- Change value of square
- Discuss how some values for the square would not work (cents" remainders)

Related Tasks

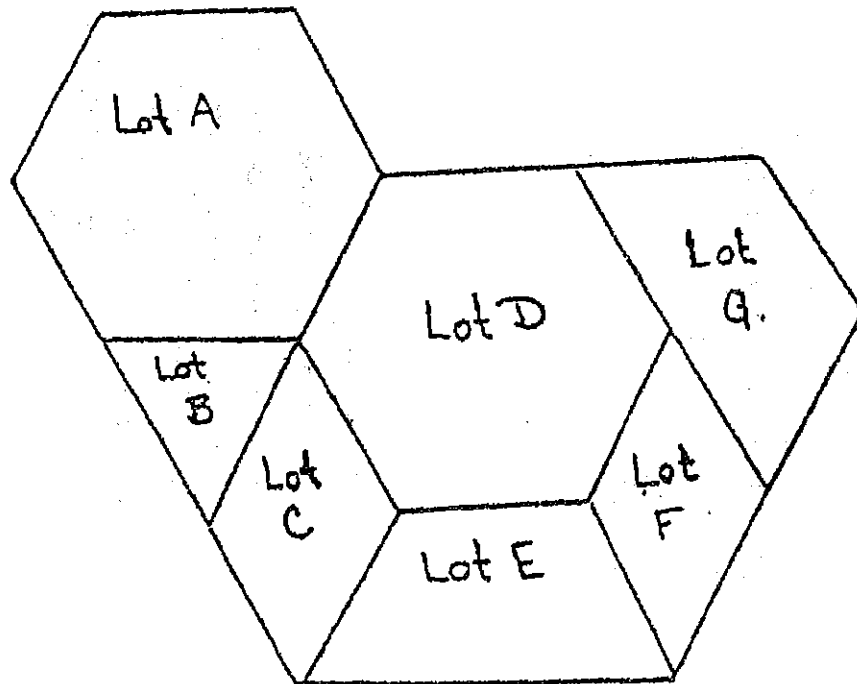
See Resource Binder

P/S	R/P	Com	Con	Rep	AC/LV

Name _____

Feverish Freddy

Freddy, a very precise real estate appraiser, was sent to appraise some lots on a local property. Freddy appraised Lot A for \$88,000 but had to go home because he was not feeling well. Freddy is not able to appraise the fair market value for the rest of the lots. Help Freddy finish his appraising work. What are the fair values of all the lots? Show your mathematical thinking.



Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task (4)

Number and Operations - Fractions
Geometry

Title of Task Feverish Freddy

Content Strand(s) Addressed Geometry

Program Link

State Standard(s) Addressed

Common Core Standard(s) 4.NF.2, 3 and 4.G.3

Underlying Mathematical

Concepts

Fractional part of a whole

congruency

addition/division

money notation

number sense to \$88,000.00

Problem Solving

Strategies/Representation

model (manipulatives)

diagram/key

table

chart

Mathematical Language

model

diagram

key

table

chart

hexagon

trapezoid

rhombus

triangle

money notation

\$.

value

fractions $\frac{1}{2}$...

decimals .50...

percents 50%...

area

total

congruent

numerator

denominator

cost

profit

>

Possible Solution(s)

The Properties

Lot	name of shape	value of each lot
A	hexagon	\$88,000.00
B	triangle	\$14,666.60
C	rhombus	\$29,333.30
D	hexagon	\$88,000.00
E	trapezoid	\$44,000.00
F	rhombus	\$29,333.30
G	trapezoid	\$44,000.00

Answer See value column

Connections

- name each shape
- Show fractional/decimal percent for each shape's value
- The total of all lots is \$337,333.20
- Relate to a similar problem and state math
- Shapes are the pattern blocks
- Explain why square can't be used
- compare area and congruency

Related Tasks

See Resource Binder



GRADE 4 MATH: FARMER FRED

SUPPORTS FOR ENGLISH LANGUAGE LEARNERS

Title: Farmer Fred

Grade: 4

Linguistic Access:

In the tasks presented, we can distinguish between the vocabulary and the language functions needed to provide entry points to the math content. These vocabulary words and language functions need to be explicitly taught to ensure comprehension of the tasks. One way this can be done is by using the following approach:

- **Introduce the most essential vocabulary/language functions before beginning the tasks.**

Don't overwhelm students with too many words or concepts. Pick what is absolutely essential in each task. We suggest the following:

1. Vocabulary words/phrases:

Tier I (non-academic language): *field, worth, properties, lots, earn, removing, reward, cost, value, design*

Tier II (general academic language): *"Show your mathematical thinking"*

Tier III (math technical language): *twelve hundred dollars, fraction of the total value, fair value, polygon, symmetry, congruent, symmetrical, proportional, trapezoid, rhombus*

2. Language functions: *show, explain*

Pronounce each word for students, and have them repeat after you.

Content Access:

To provide content access to ELLs, it is important that they are familiar with the concepts of fractional parts of a whole. ELLs should already be familiar with the properties of hexagons, trapezoid, rhombus and triangles. ELLs might need to review how to assign money value to "fractional piece" and then use proportional thinking to figure out the value of other piece in order to calculate the value of a set of parts and the whole.

Also, in order to successfully engage in the mathematical tasks included in this grade level, ELLs will need to be familiar with money notation.

It would be very helpful for ELLs to have access to actual pattern blocks. This would provide them with a tactile experience that will help them make sense of the situation.





Familiarizing ELLs with Pattern Blocks

Provide ELLs with pattern blocks and engage them with these questions to allow them to make sense of the pattern blocks:

1. Identifying the different geometry figures represented by the blocks:
 - a. What geometric figure does the yellow block represent?
 - b. What geometric figure does the red block represent?
 - c. What geometric figure does the blue block represent?
 - d. What geometric figure does the green block represent?
2. Encourage ELLs to figure out the relationship between the blocks. You might frame their work by asking guiding questions, such as:
 - a. How many red blocks can you fit in the yellow block?
 - b. How many green blocks can you fit in the blue block?

It is important that they figure out all possible relationships.

3. Let the yellow hexagon stand for the unit (a cookie). Each time cookies are shared, they must be shared equally. Act out each situation described below. Look for patterns or generalizations that come from this experience.

How much does each person get if:

- a. one cookie is shared by three people.
- b. One cookie is shared by six people.
- c. Three cookies are shared by two people.
- d. Four cookies are shared by six people.
- e. Two cookies are shared by four people.
- f. Five cookies are shared by three people.
- g. Seven cookies are shared by six people.
- h. Eight cookies are shared by three people.

Possible Scaffolds and Resources:

1. Organize tasks to maximize opportunities for ELLs to engage in math discourse. Therefore, it is recommended that:
 - Teacher allows students to work collaboratively in pairs or triads and to justify their decisions to peers.

- Teacher allows ELLs to use their language resources, including their native language, gestures, drawings, etc. to convey their understandings.
 - Teacher models not only the math content but also the desired academic language in context to develop students' second language.
 - Teacher uses paraphrases and “re-voicing” (reformulation of students' statements using appropriate math terminology or syntax).
 - Teacher uses physical objects to facilitate students talk (e.g., manipulatives).
 - Teacher gives appropriate wait time for ELLs to respond.
 - Teacher gives the opportunity for ELLs to clarify their statements using different expressions.
2. Facilitate a metacognitive approach to reading a math problem by encouraging students to monitor their understanding of the situation presented. This might include asking ELLs to:
- Listen to the problem being read.
 - Read the problem by themselves or in small groups.
 - Underline relevant information.
 - Identify what the problem is asking and what they need to do to solve it.
3. In the Instructional Supports section on page 41, there are some instructional implications that teachers may use to scaffold ELLs' knowledge while introducing the concepts in the tasks. Although several are listed, we recommend the ones below, which have been slightly modified:
- Review needed academic language and model the meanings of the words/phrases, if needed (e.g., *congruency*, *equivalent*, *polygon*).
 - Make student's thinking public while solving a problem in order to provide ELLs with possible entry points to task.
 - Use manipulative to investigate fractional part of a whole-fraction bars and circles.
 - Show and make reference to a number line.



GRADE 4 MATH: FARMER FRED

SUPPORTS FOR STUDENTS WITH DISABILITIES

Instructional Supports for Students with Disabilities using UDL Guidelines

Background Information

- ***Provide options for language, mathematical expressions, and symbols. Clarify syntax and structure.***

Learners have varying degrees of understanding linguistic and non-linguistic representation. The manner in which vocabulary, mathematical expressions and symbols are communicated should be varied so that students may experience multiple entry points to comprehend and learn concepts. It is critical that students not only have access to grade level mathematics but that they understand and are clear on what is being taught. Although there is not just one way to solve the Farmer Fred Assessment Task, students must be able to utilize a number of math skills and comprehend several underlying mathematical concepts and vocabulary to solve this multi-step problem. To motivate students, guide them in making mathematical connections and involve them in their learning. Provide opportunities to integrate math, technology and art. Hands-on experiences for students such as using manipulatives and math software will support them in grasping key concepts. Ensure that students have equal access to information by providing alternative representations to make more explicit the relationship between elements of meaning.

The strategies and the instructional supports provided below are intended to anchor students' mathematical learning and thinking. They are designed to provide insight as to how you can use UDL guidelines on the Farmer Fred Assessment Task and other instructional tasks in the unit. Students need multiple exposures and opportunities to practice the underlying mathematical concepts and language that is presented in this Farmer Fred unit.

Pre-Activities:

- ***Activate or supply background knowledge.***

For students to solve the Farmer Fred Assessment Task they need to make mathematical connections by understanding and applying certain underlying mathematical concepts, mathematical language and problem solving strategies (see Preliminary Planning Sheet below). Although we are providing examples for only a few of the mathematical concepts, problem solving strategies, and mathematical language terms, similar strategies and approaches can be applied to facilitate the learning of those concepts, problem solving strategies and terms not provided.

Farmer Fred Unit Planning Guide

Underlying Mathematical Concepts	Mathematical Language	Problem Solving Strategies and Representation
Congruency Fractional parts of a whole Properties of a hexagon, trapezoid, rhombus, and triangle Money notation Symmetry Patterns Mathematical computation Fair value Number sense up to \$1200	Shapes -- triangle, hexagon, trapezoid, rhombus, Money notation Congruent Symmetrical Total Diagram Table Numerator Denominator Similar Fractions to whole Fair value Equivalent	Model Manipulatives – pattern blocks, fake money Diagram Key Chart (table) Guess and check

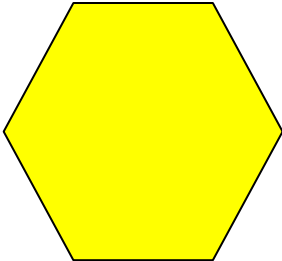

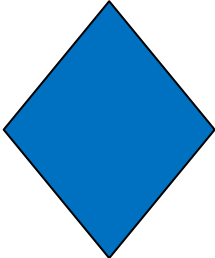
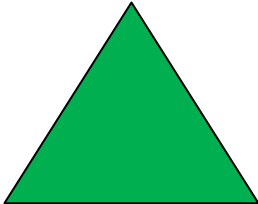
Teach/Review Underlying Mathematical Concepts and Language

As previously mentioned, it is important to ensure that students have access to different forms of representation both linguistic and non-linguistic. This aids in clarity and comprehensibility across all learners. To be successful in the Farmer Fred Assessment Task, students will need to understand all underlying mathematical concepts and language. Promoting connections to students' experiences and prior knowledge will facilitate their learning.

Pre-Activity #1

- ***Customize the display of information.***

On a document camera, white board (Microsoft Word Shapes), using large cut-outs of construction paper shapes or using actual pattern blocks, elicit from students the names of each of the shapes (hexagon, trapezoid, rhombus and triangle) and the number of sides each shape has. On the chart below, record students' responses. This can be used as a reference for future activities.

Shape	Name	Number of Sides
	hexagon	6
	trapezoid	4
	rhombus	4
	triangle	3

Pre-Activity #2

- ***Use multiple tools for construction and composition.***

Distribute pattern blocks to pairs of students that include one hexagon, two rhombi, two trapezoids, and six triangles. Have students find all the different ways to put the shapes together to make ONE hexagon. Use the worksheet below to complete this activity. Point out to students that they will see the word “rhombi” in the worksheet. Elicit from students what rhombi are. To quickly facilitate this, show students the difference with pattern block rhombi.

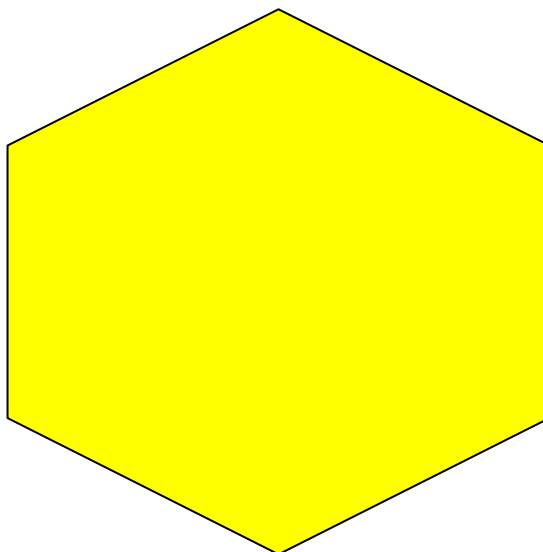
Group Name: _____

Date: _____

Students' Names: _____

ONE Hexagon Equals...

With a partner, use as many pattern blocks as you need to make one hexagon. Record your responses to the questions below.



1. One hexagon is made up of _____ trapezoids.
2. One hexagon is made up of _____ triangles.
3. One hexagon is made up of _____ rhombi and _____ triangles.

- ***Foster collaboration and communication***

Have students share their findings for each question on the worksheet. Have students discuss the relationship between the shapes.

- ***Increase mastery-oriented feedback.***

Provide students with verbal and/or written feedback that is substantive and informative rather than comparative and competitive as a class (i.e., share out) , small groups (i.e., guided math groups) or individually (i.e., conferencing).

Pre-Activity #3

- ***Provide options for language, mathematical expressions, and symbols.***

On the Farmer Fred Assessment Task field, students have to eventually determine fair value for and the worth of each field. To determine fair value and worth, students must be able to recognize that the hexagon-shaped fields, the trapezoid-shaped fields, etc. are congruent. Therefore, introduce the mathematical concept of congruency—same shape and same size (in any direction). Demonstrate using the pattern blocks, how the hexagons are congruent (this can be continued for each of the shapes—trapezoid, rhombus and triangle). You may use the worksheet below to reinforce the concept of congruency for the students. Students will identify whether the shapes are congruent or not. Again, students may work in pairs to complete this worksheet. Instead of the worksheet below, you may develop math card games (concentration, pair it) for students to practice the concept of congruency.

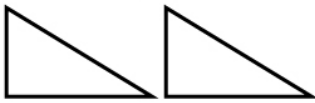
Group Name: _____

Date: _____

Students' Names: _____

Congruent Shapes

CONGRUENT figures are the same size and shape.



CONGRUENT



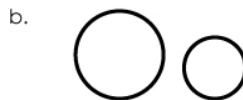
CONGRUENT

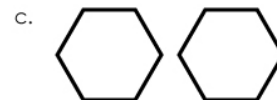


NOT CONGRUENT

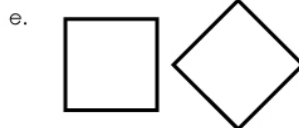
Write CONGRUENT or NOT CONGRUENT for each.





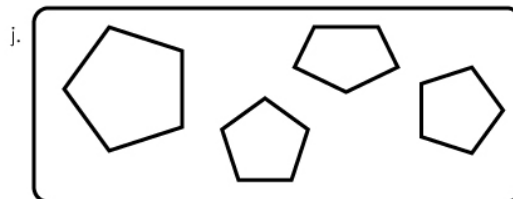
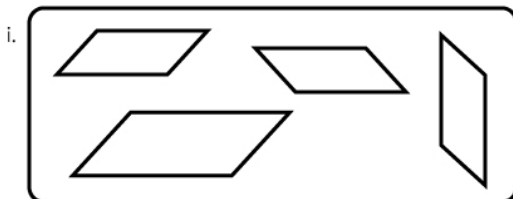
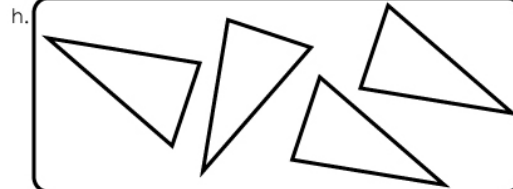
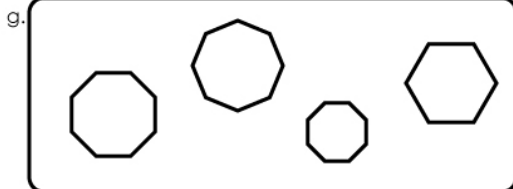








Color all the congruent shapes in each box.

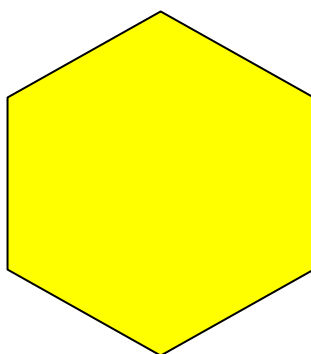


From: www.superteacherworksheets.com

Pre-Activity #4

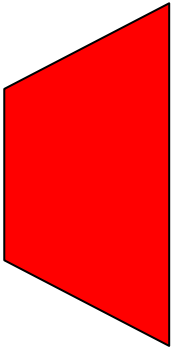
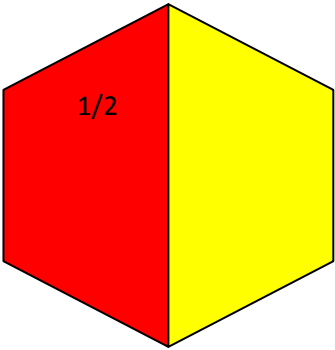
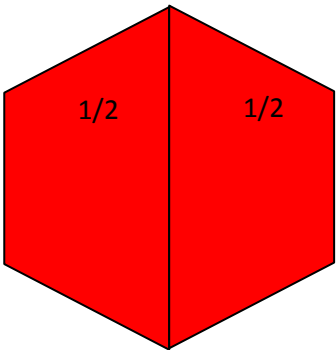
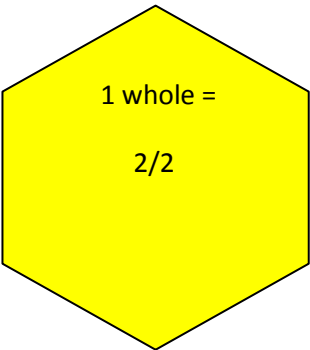
- ***Maximize transfer and generalization.***

Fractions are another mathematical concept that students will need to master to successfully complete the Farmer Fred task. When we talk about fractions, we talk about PARTS of a WHOLE. The WHOLE can be divided into EQUAL parts. Refer students to the worksheet they completed entitled, “One Hexagon Equals...”. Using a white board (can use Microsoft word shapes), document camera, overhead or chart paper, model and discuss that a hexagon equals one whole.



- ***Guide information processing, visualization, and manipulation.***

Elicit from students how many trapezoids are in one hexagon. Place the trapezoids over the hexagon. Model and think aloud for students that two trapezoids equal one whole hexagon and that each trapezoid is one half of the whole (i.e., $\frac{1}{2}$). Continue the same process with each of the other shapes (6 triangles = 1 hexagon [1 triangle is $\frac{1}{6}$ of the whole], one rhombus = $\frac{1}{3}$ of a hexagon [you will need place two rhombi in the hexagon and two triangles to demonstrate this concept. In addition, students will need to see that two triangles equals one rhombus]. A chart can be created by students for reference tracing pattern blocks or cut-outs. A jump-start sample is provided below.

Fractional Parts	Equals	Whole
 <p>1 trapezoid</p>	<p>=</p>	 <p>1/2 hexagon</p>
 <p>2 trapezoids</p>		 <p>1 hexagon</p>

Introduce the FARMER FRED Assessment Task

- ***Use multiple media for communication.***

Using the melody to ***Old McDonald Had a Farm***, engage students in singing the song reviewing the shapes introduced in the Pre-Activities. This can also be performed using props (i.e., students can dress up as farmers, use construction paper shapes, etc...) and/or showing the Farmer Fred field. Let students know that in the problem you will be solving the Farmer's name is Fred and that the song reflects his "nick name" (we named him "Freddy" for the sake of singing the correct meter in the song)!

Farmer Freddy Had a Farm

Farmer Freddy had a farm

E-I-E-I-O

And on his farm he had eight fields

E-I-E-I-O

With a hexagon field here and a hexagon field there,
here a hexagon, there a hexagon everywhere a hexagon

Farmer Freddy had a farm

E-I-E-I-O

Farmer Freddy had eight fields

E-I-E-I-O

With a trapezoid field here and a trapezoid field there,
here a trapezoid, there a trapezoid everywhere a trapezoid

Farmer Freddy had a farm

E-I-E-I-O

Farmer Freddy had eight fields

E-I-E-I-O

With a rhombus field here and a rhombus field there,
here a rhombus, there a rhombus everywhere a rhombus

Farmer Freddy had a farm

E-I-E-I-O

Farmer Freddy had eight fields

E-I-E-I-O

With a triangle field here and a triangle field there,
here a triangle, there a triangle everywhere a triangle

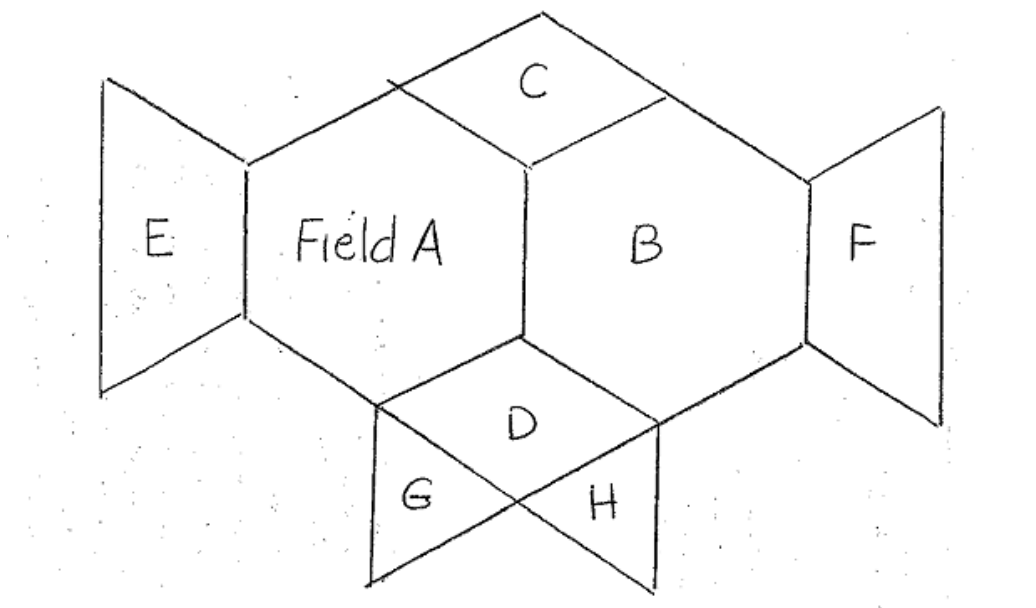
Farmer Freddy had a farm

E-I-E-I-O

Farmer Freddy had a farm
 E-I-E-I-O
 And on his farm he had eight fields
 E-I-E-I-O
 With a hexagon field here and a trapezoid field there,
 here a rhombus, there a triangle, everywhere a different field
 Farmer Freddy had a farm
 E-I-E-I-O

- **Highlight patterns, critical features, big ideas, and relationships.**

Read the Farmer Fred Assessment Task aloud. Use the Farmer Fred field as a visual as you facilitate students' understanding the key concepts and language in this task and the problem that needs to be solved. Recreate Farmer Fred's field with a document camera; students can place the appropriate pattern blocks in the exact layout as Farmer Fred's farm. You may also use a white board— *using programs such as **Computer-Aided-Design** or **Kidspiration***—an overhead transparency, or chart paper. Model thinking aloud by using pattern blocks and dividing the hexagon into fractional parts with the remaining shapes (trapezoids, rhombi and triangles) to enable students to understand the problem. You can hand out adhesive dots or labels to have students label each of the pattern blocks with the appropriate fraction (e.g., a trapezoid is $\frac{1}{2}$ of a hexagon, a triangle is $\frac{1}{6}$ of a hexagon, a rhombus is $\frac{1}{3}$ of a hexagon).


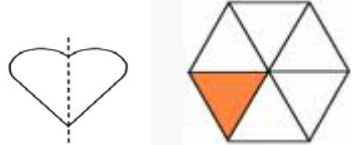
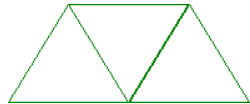



- **Clarify vocabulary and symbols.**

Using student-friendly definitions, explain concepts such as fair value, properties, etc. For example in the sentence, “The fields are formed with the same properties as your pattern blocks,” you might say, “In other words, the fields match your pattern blocks—just as you saw in the activities that you have completed up to this point.” Also, explain concepts such as fair value by relating it to real life and having students act it out or role play. You can have large pre-cut construction paper shapes of Farmer Fred’s field (rhombi, hexagons, trapezoids and triangles) that the students can hold and assemble into the eight fields represented in the Farmer Fred task. Guide students in restating the problem to ensure that students understand what is being asked of them. For example, a student may rephrase part of the problem in their own words such as, “I need to find out how much and what fraction each field is worth.”

- **Offer alternatives for auditory information.**

Provide a reference for students by using visuals and realia to create a chart of vocabulary and mathematical concepts on chart paper, white board, or overhead with student-friendly definitions. You may refer to the following websites for student-friendly definitions: (<http://www.ldoceonline.com/dictionary> and <http://www.math.com/school/glossary/glossindex.html>). Google Images is an excellent resource for visuals. Students may also create a math dictionary or reference guide that models the vocabulary chart below to promote independent learning and self-monitoring.

field	an area of land in the country, for example, where crops are grown	
fraction	part of a whole number, such as $\frac{1}{2}$ or $\frac{1}{6}$	
congruent	the <u>same shape</u> AND the <u>same size</u> (i.e., these 3 triangles are congruent)	
$\$1200 =$ $\$1,200$	twelve hundred dollars = one thousand two hundred dollars	

- ***Illustrate through multiple media.***

The concept of \$1200 can also be shown to students on a white board (or document camera or chart paper) and by using fake money. You can make copies of the hundred dollar bill below for your students.



- ***Maximize transfer and generalization.***

Students can practice counting by hundreds, “One hundred, two hundred, three hundred,” until they get to twelve hundred so they get a better sense of what makes up \$1200.

General Problem Solving Strategies

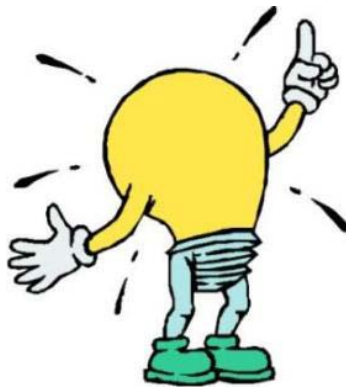
The Farmer Fred Assessment Task, along with the instructional support tasks included in this unit facilitates the teaching of mathematical concepts and topics through problem solving contexts. It is in this way, students will be able to develop a deep understanding of the topics being taught. This approach engages students in doing mathematics and in developing mathematical thinking. The problem solving process needs to be modeled and practiced until students are able to do it independently. Students need to learn that there is more than one way to solve a problem.

Learning how to solve problems in mathematics requires students to be able to read and understand the mathematical language and concepts. They need to be able to identify what they need to do/or find out, what information the problem gives them and whether the problem has all the necessary information. They then need to make a plan of action. They can use manipulatives, draw pictures, look for patterns, make a chart, act it out or talk it out. Finally they would solve the problem, check to see if their answer makes sense and explain their thinking. Included below is a 4-step problem solving procedure to facilitate the aforementioned.

- **Support planning and strategy development.**
- **Enhance capacity for monitoring progress.**

Introduce a 4-step procedure for solving a mathematical problem. **Read and Understand, Plan, Solve and Look Back/Reflect.** These steps can be entered into the math journal for students' reference and posters can be displayed around the room to serve as visual prompts.

STEPS FOR SOLVING A MATH PROBLEM



1. **READ and UNDERSTAND**
2. **PLAN**
3. **SOLVE**
4. **LOOK BACK/REFLECT**

- ✓ Guide students in understanding what good problem solvers do when they approach a word problem. Model thinking process:

Step 1

Boys and girls, to **Read and Understand**, a good problem solver...

- ✓ Highlights or circles key words and numbers in the problem

For example:

Farmer Fred's fields are worth **twelve hundred dollars** total. The fields are formed with the **same properties** as your **pattern blocks**. **Each field's value** is based on its **size**. **What fraction of the total value is each field worth?** **How much is each field worth?** Show and explain all of your mathematical thinking.

- ✓ Re-reads the story problem as many times as necessary to understand.
- ✓ States in his/her own words what questions need to be answered.

Step 2

Boys and girls, to **Plan**, a good problem solver...

- ✓ Chooses the strategy or strategies that will help solve the problem quickly and easily.
(Draw a picture, use manipulatives, make a model, create a graph or chart, guess and check, look for a pattern, and/or write an equation.)

FIELD	FRACTION	VALUE
A		
B		
C		
D		
E		
F		
G		
H		

Step 3

Boys and girls, to **Solve**, a good problem solver...

- ✓ Labels the math work in the solution area.
- ✓ Checks to see if he or she is following his or her plan.
- ✓ Works carefully to see that he or she does not make mistakes.

Step 4

Boys and girls, to **Look Back/Reflect**, a good problem solver...

- ✓ Checks to see if he or she has answered all the questions and if the answers are reasonable.
- ✓ Checks to see if everything is labeled.
- ✓ Checks computation for accuracy.
- ✓ Is able to explain how his or her answer is reasonable and correct.
(Before writing, make a word “splash” or word bank of math vocabulary related to the problem, in the margin beside the written response area.)

PROBLEM SOLVING PLANNER		
Read and Understand	Plan	Solve
What am I being asked to solve?	What strategy or strategies can I use to answer the question or questions?	Show and label ALL my work.
Look Back/Reflect - Does my answer make sense? Explain your mathematical thinking.		

Additional Resources

Books: The shape and other stuff by Dr. Seuss
The shape of things by Dayle Ann Dobbs

Websites:

www.mathleague.com

www.mathforum.org

<http://www.learninggamesforkids.com/4th-grade-math.html>

<http://www.math.com/school/glossary/glossindex.html>

<http://www.idoceonline.com/dictionary/fraction>

www.superteacherworksheets.com